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PREFACE

This volume of the "Verslagen en Verhandelingen" ("Reports and Transactions") of the "Nationaal Lucht- en Ruimtevaartlaboratorium" ("National Aero- and Astronautical Research Institute")* at Amsterdam, the Netherlands, contains the report No. TR-M. 2070 of the Materials and Structure Section of the laboratory.

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July 1961,
Amsterdam.

A. J. Marx

Director of the Nationaal Lucht- en
Ruimtevaartlaboratorium

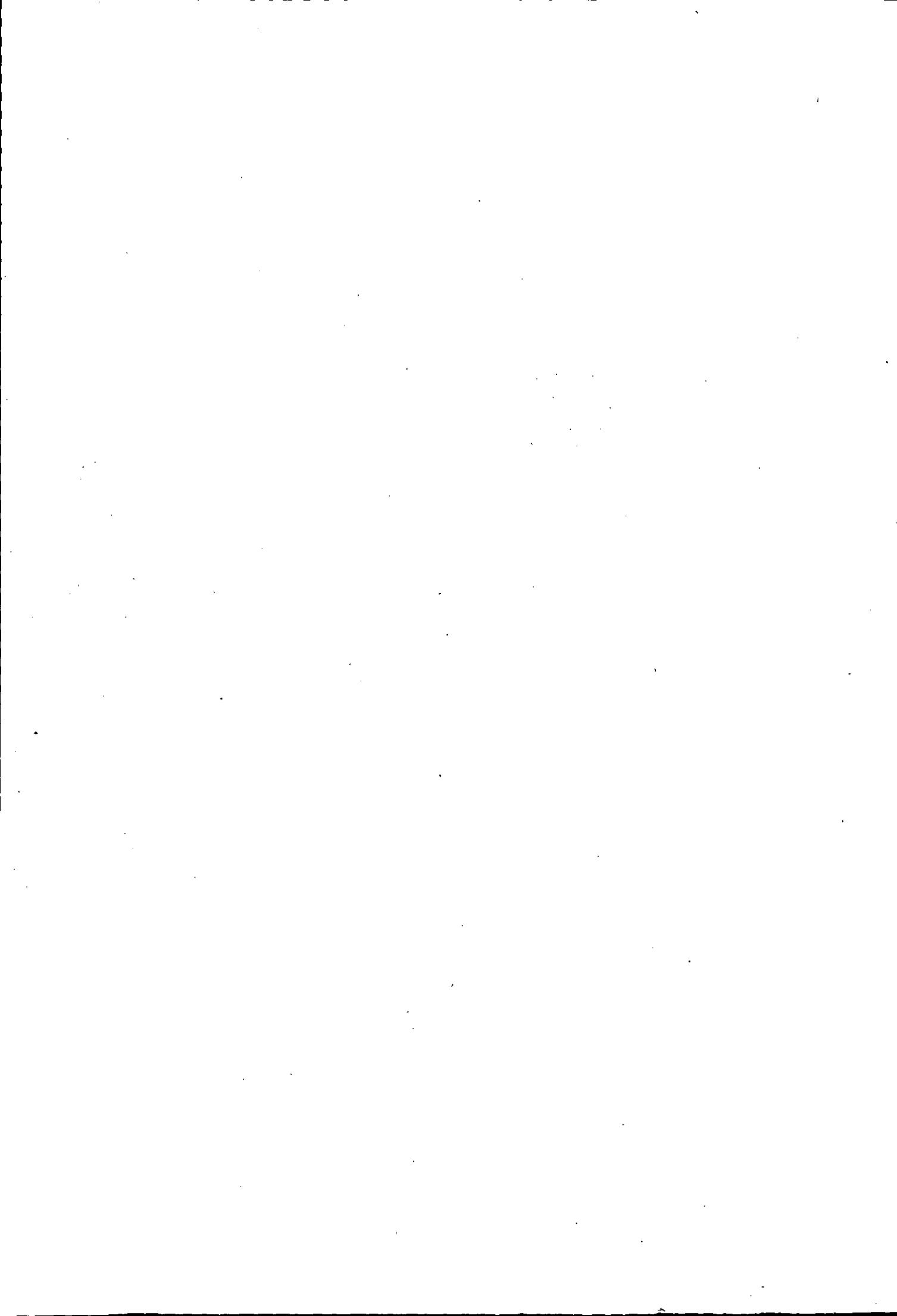
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TECHNICAL REPORT M 2070
PROGRAM-FATIGUE TESTS ON NOTCHED LIGHT ALLOY
SPECIMENS OF 2024 AND 7075 MATERIAL

BY

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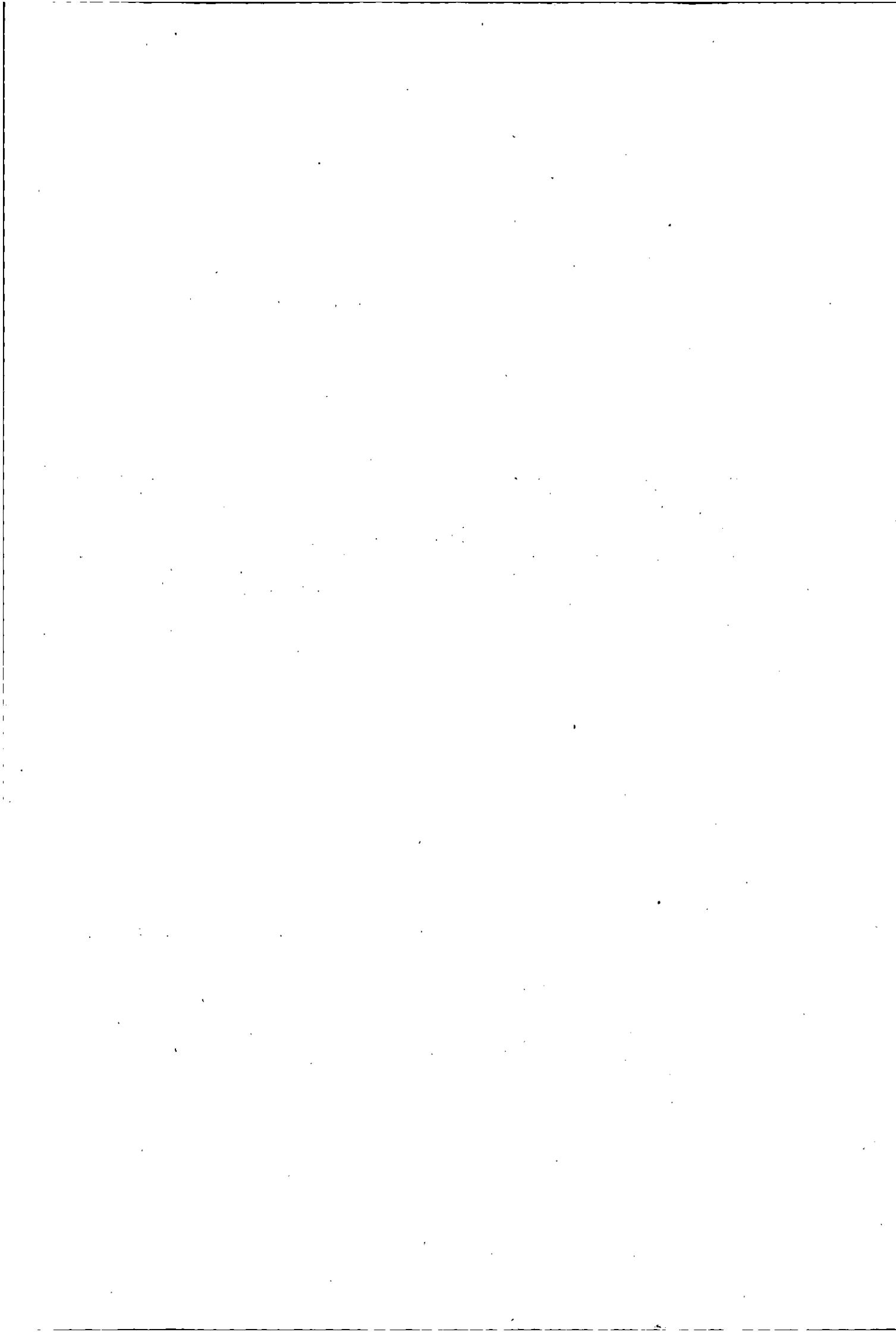
J. Schijve and F.A. Jacobs

SUMMARY

Program-fatigue tests and constant-amplitude tests at positive mean stresses have been performed on 2024 and 7075 riveted joints in order to study the effect on the parameters of the program, the scatter under program-fatigue loading and the effect of occasional very high loads, and to check the validity of the Palmgren-Miner rule.

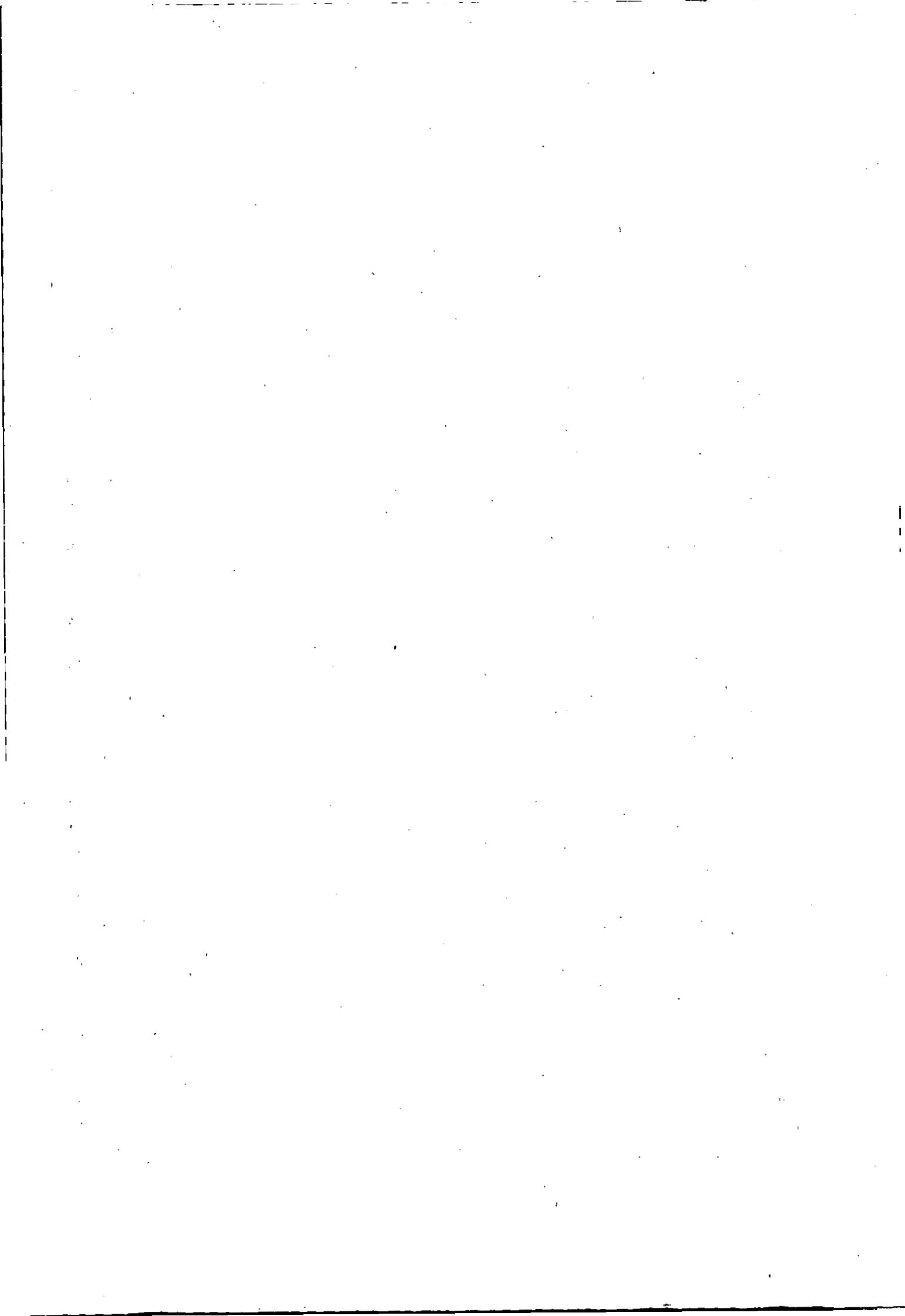
The results of similar investigations on notched light alloy specimens or structures have been summarized. A discussion is presented on the above mentioned objectives and the following questions: What is the usefulness of the Palmgren-Miner rule for the designer? How good is program-fatigue testing as a representation of the load-time history in service? What information may be expected from program-fatigue testing and how does it compare with the information of alternative methods?

This investigation has been performed under contract with the Netherlands Aircraft Development Board (N.I.V.).



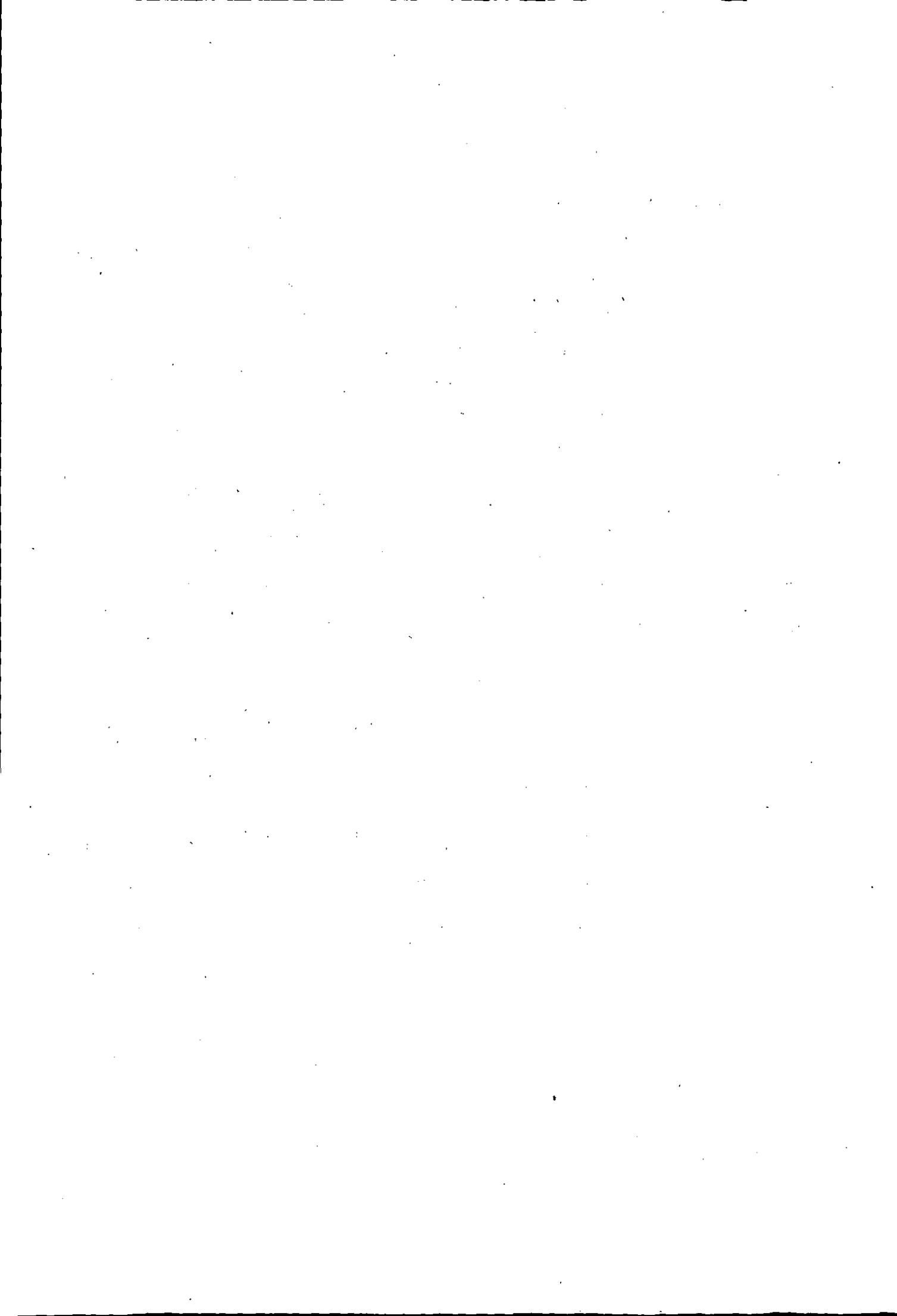
Contents

Nomenclature, symbols and units.	3
1 Introduction.	9
2 Specimens and testing procedures.	10
3 The constant-amplitude fatigue tests.	11
4 Loading programs for the program-fatigue tests.	11
5 Results of the program-fatigue tests.	12
6 Review of the results of program-fatigue tests of other investigations.	12
6.1 Scope of the chapter.	12
6.2 Results of investigations on small specimens.	12
6.3 Results of investigations on structures.	14
6.4 Results of tests with very high loads.	15
6.5 Results of random-noise fatigue tests.	15
7 Discussion.	16
7.1 Scope of the chapter.	16
7.2 The effect of different parameters of the program-fatigue test on the endurance.	16
7.2a The load sequence in a period.	16
7.2b The length of the period.	16
7.2c The lowest S_a to be included in the program.	17
7.2d The highest S_a to be included in the program.	17
7.2e The number of load levels.	17
7.3 Periodic high loads and ground-to-air cycles.	18
7.4 The validity of the Palmgren-Miner rule.	19
7.4a Small specimens.	19
7.4b Structures.	20
7.5 Differences between program-fatigue testing and service loading.	20
7.5a The effect of the rate of loading.	21
7.5b The effect of rest periods.	21
7.5c The environmental effect.	21
7.5d The load sequence in a program-fatigue test and in service.	22
7.5e Recapitulation of the value of program-fatigue testing as a means of simulating service loading.	23
7.6 The value of the Palmgren-Miner rule for the designer.	24
7.7 The value of program-fatigue testing for the designer.	25
7.8 Scatter.	27
8 Conclusions.	28
9 List of references.	32
6 tables	
53 figures	



Nomenclature, symbols and units.

- Palmgren-Miner rule** : Rule for life predictions for a fatigue loading with a variable load amplitude. It is written as $\sum \frac{n_i}{N_i} = 1$ or $\sum \frac{n}{N} = 1$. n_i is the number of load cycles applied at the load amplitude (i) and N_i is the corresponding endurance at that load amplitude if it is applied as a constant-amplitude loading. The rule states that failure will occur at the moment that the summation of the contribution $\frac{n_i}{N_i}$ at all load amplitudes is equal to one. The rule does not consider scatter. It was first published by Palmgren (ref. 36). It is, however, better known from a publication by Miner (ref. 34).
- Program-fatigue loading** (or program loading) is a fatigue loading with a periodic variation of the magnitude of the load amplitude. The variation may be a continuous one or it may involve a number of discrete values (the more usual case).
- Program-fatigue test** (or program test): Fatigue test with a program-fatigue loading. In the literature it is also called: Spectrumtest and service-endurance test.
- Period** : The period of a program-fatigue test is the period of the variable amplitude, see fig. 1.
- Period length** : Number of cycles in one period.
- Constant-amplitude (-fatigue) test** : Fatigue test, with a constant load amplitude, i.e. a conventional fatigue test.
- RAE-test** : A fatigue test on an almost complete aircraft structure or large component (e.g. wing), involving gust loads with a constant amplitude, landing loads and pressurisation loads on the fuselage (if applicable).
- Random-load test** : Fatigue test with an irregularly varying load.
- Randomized-program-fatigue test** : Program-fatigue test, in which the batches of load cycles with the same amplitude within a certain period are applied in a random order.
- Load spectrum** : Statistical distribution function of load amplitudes.
- Peak-method** : Method for reducing a load-time history to a load spectrum, which is based on the peak-loads, i.e. the maxima and minima on the time history, see section 7.5d.
- Range-method** : Method for reducing a load-time history to a load spectrum, which is based on the load ranges, i.e. the differences between successive maxima and minima of the time history.



- A = cross sectional area.
 k_t = theoretical elastic stress concentration factor.
 m = number of times that a certain gust velocity is exceeded in a certain flight distance.
 n = number of load cycles at a certain stress level.
 N = fatigue life (= endurance) under constant-amplitude loading.
 N' = fatigue life (= endurance) under program-fatigue loading.
 P = load.
 P_a = load amplitude.
 P_m = mean load.
 P_u = ultimate design load.
 S = stress.
 S_a = stress amplitude.
 S_E = fatigue limit = endurance limit.
 S_m = mean stress.
 S_{max} = maximum stress of a load cycle or maximum stress applied in a program-fatigue test.
 S_{min} = minimum stress of a load cycle or minimum stress applied in a program-fatigue test.
 S_u = ultimate tensile strength or stress at P_u .
 $S_{0.1}$ = yield stress for 0.1 % permanent set.
 $S_{0.2}$ = yield stress for 0.2 % permanent set.
 U = gust velocity.
 α, β = constants of Weibull's S-N-formula.
 δ = elongation for a gage length of 2" (~ 51 mm).
 δ_5 = elongation for a gage length of 5ϕ or $5.65\sqrt{A}$.
 δ_{10} = elongation for a gage length of 10ϕ or $11.3\sqrt{A}$.
 ϕ = diameter.

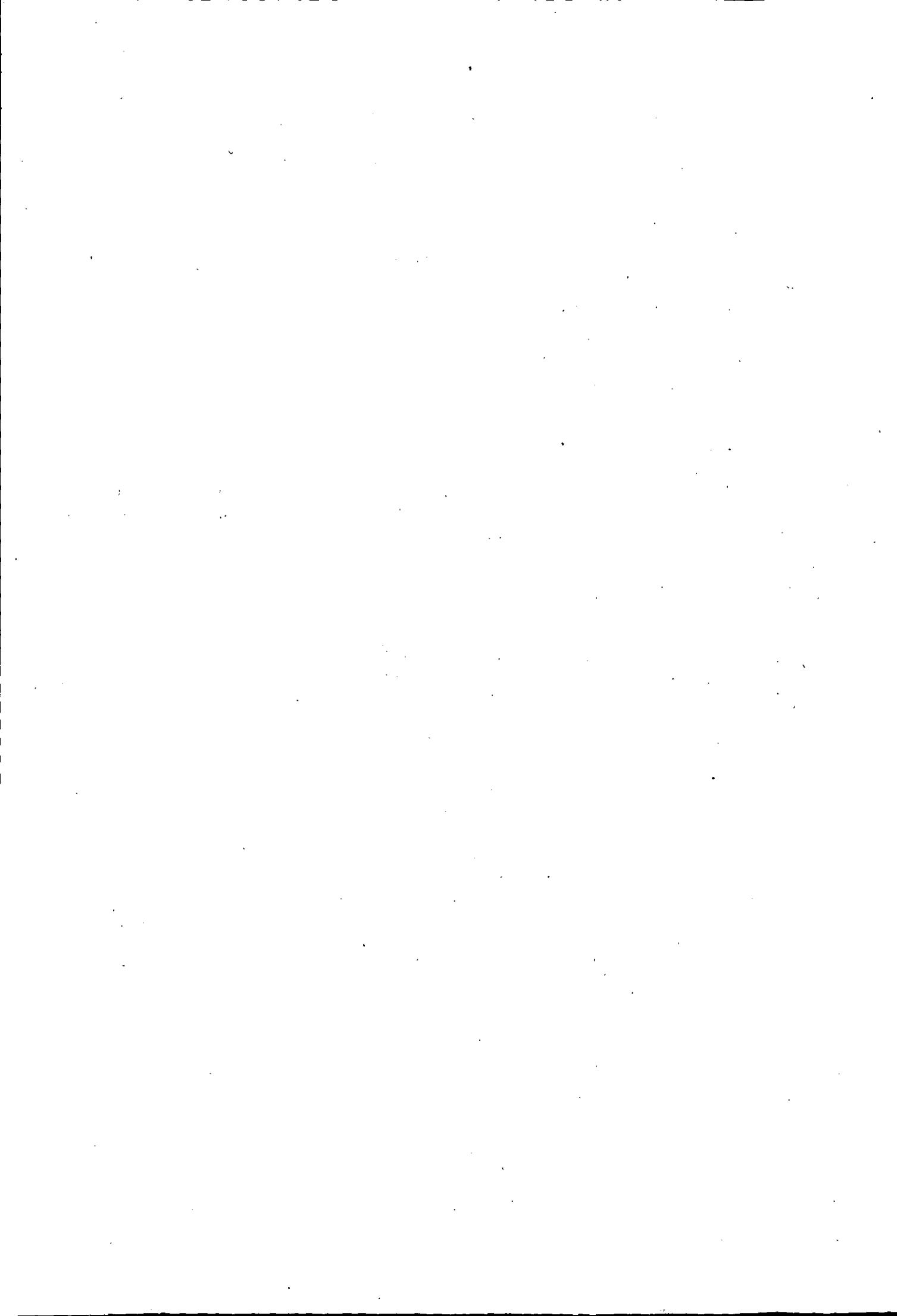
Units: Dimensions in mm. 1 mm = 0.04 inch, 1 inch = 25.4 mm.

Stresses in $\text{kg/mm}^2 = 1,422 \text{ p.s.i.} = 0.635 \text{ t.s.i.}$

$1000 \text{ p.s.i.} = 0.703 \text{ kg/mm}^2$.

$1 \text{ t.s.i.} = 1.574 \text{ kg/mm}^2$.

Number of cycles : 1 kc = 1000 cycles.



1. Introduction

In a previous report (ref. 45) the available literature on cumulative-damage tests, i. e. fatigue tests with a variable load amplitude, has been reviewed. The conclusion was drawn that the cumulative-damage behaviour of notched specimens could be significantly different from the behaviour of unnotched specimens. This implies that the usefulness of the Palmgren-Miner rule, $\sum \frac{n}{N} = 1$, as far as it concerns the designer, should be judged from the results of tests on notched specimens only. Results of such tests gave the impression that life predictions based on this rule would, on the average, be on the safe side. In order to give more empirical justification to this statement a large number of program-fatigue tests have been carried out. A riveted lap joint has been chosen as the specimen, since in practice many of the fatigue cracks originate from rivet holes. The tests were performed at a positive mean stress and under different types of program-loading, which were associated with a gust spectrum. It was felt that much of the fatigue trouble in practice had occurred under such conditions. As materials 2024 and 7075 have been used. The objectives of the test-program were:

- (a) To check the validity of $\sum \frac{n}{N} = 1$ for a riveted joint at a positive mean stress under a number of periodic load sequences associated with a gust spectrum.
- (b) To compare the behaviour of 2024 and 7075 specimens.
- (c) To study the scatter in program-fatigue tests as compared with the scatter in constant-amplitude fatigue test.

In the design stage of an aircraft it will be necessary to make life estimates for certain components. For this purpose the Palmgren-Miner rule is widely used, the calculations being based on more or less representative S-N-curves. In the final stage of a design it may be desirable to dispose of a more accurate life estimate. One may then decide on different procedures, for instance:

- (1) A number of components or structures are tested to determine the relevant S-N-curve. The fatigue life is then calculated employing $\sum \frac{n}{N} = 1$.
- (2) Program-fatigue tests are carried out on one or more components or structures, the load sequence being in accordance with the anticipated load spectrum in service.
- (3) A third method is the so-called RAE-test (ref. 1). This method is intermediate between the two other methods. In general it is performed on a complete structure and the loadings include at least gusts, ground-to-air cycles and pressurization cycles on the fuselage. The gust loading, however, is applied as a constant-amplitude loading.

These methods all have their merits and drawbacks. To arrive at a technical statement with regard to a choice between them, it was realized that program-fatigue testing involves some inherent problems for which at least a qualitative answer had to be given. These problems are essentially concerned with the programming of the anticipated load spectrum. The load spectrum, which is the distribution function of the loadings, has to be simplified in such a way that it can be introduced in a test. Parameters to be chosen for that purpose are indicated in fig. 1 and it is to be expected that each parameter has some effect on the result of the test. An additional objective of the test program was therefore:

- (d) To study the effects of the period length, of the highest and lowest loads to be included in the program and of the load sequence in a period on the fatigue life obtained in a program-fatigue test.

In view of the predominant effect which very high and therefore rather rare loads may have on the fatigue life, some series of program-fatigue tests were performed with such very high loads included. The fifth objective was:

- (e) To study the effect of very high loads on the fatigue life in a program-fatigue test.

For the above mentioned objectives a large number of test series, each consisting of 7 program-fatigue tests, were performed in addition to constant amplitude tests for establishing the S-N data. This report gives an account of the planning of the tests, the test procedures, the results and their evaluation. A summary of the results with a brief discussion was already given in ref. 47. In addition a literature review on comparable investigations has been made. It has been tried to condense the results of other investigations in a number of figures showing the salient points. A discussion based on these figures had the following objectives:

- (f) To arrive at some guide lines with respect to systematical trends in the deviations from the Palmgren-Miner rule and at a qualitative understanding of these deviations.
- (g) To establish some guide lines with respect to planning of program-fatigue tests.
- (h) To arrive at an appreciation of program-fatigue testing as a replacement of service loading.
- (i) Finally it has been tried to give an outline of the implications for the designer of the validity or non-validity of the Palmgren-Miner rule and the value of the information to be obtained from program-fatigue tests and their alternatives, such as the RAE-test.

Acknowledgement: Advance copies of refs. 24, 25, 26 and 35 were received from Mr. Haas and Mr. Hardrath, and Dr. Gassner provided additional information for the preparation of figs. 19 to 23. The authors greatly appreciated this kind help.

2. Specimens and testing procedures.

Two sheet materials have been used. The static properties were:

Material	$S_{O.2}$ (kg/mm ²)	S_u (kg/mm ²)	Elongation ($\epsilon=2$ ")
7075 Clad	45.9	54.0	11.2 %
2024 Alclad	33.0	46.1	16.3 %

The dimensions of the specimens are shown in fig. 2. The static strength of the specimens based on the net area was 31.5 kg/mm² and 33.7 kg/mm² for the 7075 and 2024 specimens respectively. In the static tests the specimens failed by shearing of the rivets. In the fatigue tests, however, failure occurred in the sheet material. The specimens were manufactured by the Fokker Factories and were exactly similar to those tested for ref. 45. The distribution of the thickness of the specimens was approximately normal. For all test series groups of 7 or 10 specimens were drawn from the population in such a way that each group could be considered as a representative sample. Each specimen consists of two sheets which might have slightly different thicknesses. However, there was no apparent tendency to fail in the thinnest sheet, neither was there an obvious correlation between endurance and sheet thickness. It was then decided to base all calculated stresses on the net section of the sheet associated with the closing head of the rivets. The fatigue failure can propagate in one of both sheets or in both sheets at the same time. The latter occurred in 28% of all specimens. The 7075 specimens showed some preference for failure in the sheet at the side of the closing head and the 2024 specimens showed some preference for failure in the sheet at the side of the manufactured head of the rivet.

For clamping the specimens in the fatigue machine special grips were designed to allow an easy and quick exchange of specimens. The specimen and the grips are shown in fig. 3. Corrections to the indications of the dynamometer were made in view of inertia forces of the lower clamping head. At the frequency of 6000 c/min this correction was 3.3%.

In some test series compressive loads had to be applied, one at the end of each period. Buckling of the specimen was prevented by two

steel covers, see fig. 4. The inner sides of these covers had been adapted to the geometry of the specimen. To eliminate friction between the covers and the specimen felt had been bonded to the inner sides of the covers. The covers were kept on the specimens by six bolts which were hand tightened. The covers were removed from the specimen after the application of the negative loads. So, during the automatically monitored part of the program test there were no covers on the specimen.

The fatigue machine was a 10-tons Amsler Vi-

braphore, which has been described elsewhere (ref. 44). This machine is an electromagnetic resonance machine. For this test program it was equipped with a 2 tons dynamometer. The frequency was about 6000 cycles/minute. If a fatigue crack has initiated and propagated sufficiently, the resonance frequency of the vibration system decreases and the machine switches off automatically. That moment was considered as the end of the test. Although the specimen had not completely failed then, a well defined crack was clearly visible to the naked eye.

For monitoring the stress amplitude in a program-fatigue test Amsler had designed a small automatic control gear which is shown in fig. 5. A slowly rotating drum of non-conducting material and a metallic top and bottom cover is provided with vertical silver strips. A feeler which can move only vertically actuates an electromotor as soon as it is contacted by a silver strip. This motor will then move the feeler towards the free end of the strip and it stops running immediately after the feeler has lost contact with the strip. As a consequence the height level of the feeler will follow the white line on the drum in fig. 5 if the drum is rotating. The height position of the feeler is directly coupled to the photocell which is monitoring the load amplitude. Since the strips can be placed on the drum at any location and height a large variability in the programming of the load amplitude is possible. One rotation of the drum corresponds to an integer number of periods; in most test series it corresponds to one period. The rotation speed of the drum can be adjusted continuously within certain limits.

The vertical speed of the feeler, i. e. the speed of changing the load level, is about 5 mm/sec. which in this test program corresponds to a change in stress amplitude of 1 kg/mm² per second. This speed was sufficiently high for most test series, but not for all test series.

In some test series the number of load cycles at the highest stress amplitude was too small to be applied in a reasonable way with the automatic program apparatus. The desired load-sequence then had to be approximated by manual control of the apparatus. The type of approximation is shown in fig. 6.

In view of checking the correct operation of the program apparatus the load amplitude was continuously recorded by feeding a voltage of the amplifier of the fatigue machine into a recorder.

3. The constant-amplitude fatigue tests.

The mean stress for the 7075 and the 2024 specimens was 6.3 kg/mm² and 9.0 kg/mm² respectively (see chapter 4). At each amplitude 10 tests were carried out to obtain sufficiently accurate S-N-curves and an indication of the scatter. The results are presented in tables 1 and 2 and figs. 7 and 8. All specimens in one group were tested at the same load amplitude. This procedure has been adopted because in the program-fatigue tests it was more convenient to apply the same load-amplitude program throughout one test series, rather than to have the same stress-amplitude program. The latter would have been more laborious to achieve with the automatic program apparatus. The procedure implies that all specimens of a certain group were tested at slightly different stresses due to the variation in sheet thickness.

The S-N-curves of figs. 7 and 8 have been fair-
ed through the test results employing the method of least squares and Weibull's S-N-formula (ref. 57):

$$\log N = \log \alpha - \beta \log (S_a - S_E) \quad (1)$$

α , β and S_E are constants, the latter one being the fatigue limit. Numerical values for these constants are given in figs. 7 and 8. Not too much weight should be attached to the S_E values since they are largely extrapolated results.

4. Loading programs for the program-fatigue tests.

In principle all programs were based on a gust spectrum published by Taylor in 1953 (ref. 51). This spectrum is shown in fig. 9 and shows a linear relation between the gust velocity and the logarithm of the number of exceedings. It can be expressed as:

$$U = -5.625_{10} \log m + 43.75 \quad (2)$$

U is the gust velocity (ft/sec) and m is the number of times that U is exceeded in 10^7 miles of flying.

The mean stress S_m was based on the ultimate stress of the specimen S_u as follows:

$$S_m = \frac{S_u / 1.5}{n_{50}} \quad (3)$$

n_{50} is the assumed load factor corresponding to a 50 ft/sec gust. The factor 1.5 is the safety factor and $S_u/1.5$ represents the stress at limit load. For the load factors values of 3.33 and 2.5 were selected for 7075 and 2024 specimens respectively. The S_m -values obtained by eq. (3) were 6.3 and 9.0 kg/mm² respectively. The load factor for the 2024 specimens was purposely selected somewhat lower to arrive at a higher mean stress, because in the program-fatigue test the highest stress amplitude (apart from the high single loads) which could be applied automatically was equal to the mean stress. From this point of view a slightly higher mean stress was felt to be advantageous, since it allows somewhat more flexibility in the variations of the load programs. The gust spectrum was converted into a stress amplitude spectrum by means of the relations

$$S_m + S_{a50} = \frac{S_u}{1.5} \quad (4)$$

and

$$S_a = \frac{U}{50} S_{a50} = \frac{U}{50} \left(\frac{S_u}{1.5} - S_m \right) \quad (5)$$

S_{a50} and S_a are the stress amplitudes corresponding to gust velocities of 50 and U ft/sec respectively. Eq. (5) implies that S_a is supposed to be linearly related to the gust velocity up to $U = 50$ ft/sec, at which the maximum stress corresponds to the stress at limit load, see eq. (4).

In order to select the numerical values for the stress amplitudes to be used in the program-fatigue tests, the damage-distribution curves were constructed. The principle of this construction has been indicated in fig. 10. The curve shows how the damage done by the stress spectrum is distributed over all the values of the stress amplitude under the restriction that the Palmgren-Miner rule is correct. In fig. 10 the so-called level of maximum damage is indicated. This level received special attention in the literature (ref. 6). Fig. 11 shows the damage-distribution curves and the fatigue curves for both materials. In fig. 12 it has been indicated how the selection of four of the S_a -values for the program tests is related to the level of maximum damage. Higher and lower S_a -values were chosen more or less arbitrarily. All S_a -values employed are mentioned in the headings of tables 3 and 4. Fig. 13

shows how the Taylor-spectrum has been approximated employing the S_a -values from tables 3 and 4.

The numbers of load cycles per period for all stress levels are given in tables 3 and 4 and the load sequence for each test series has been indicated schematically in figs. 14, 15, 16 and 17. Each test series consisted of 7 identical tests.

The frequency of occurrence of the periodic high loads which were applied in some test series is once in a period; for one period $\sum \frac{n}{N} \sim 0.1$. It can be derived from fig. 9 that the stress amplitudes to be exceeded with this frequency are 10.12 kg/mm² and 10.07 kg/mm² for the 7075 and 2024 specimens respectively. In the test the higher values of 11.80 kg/mm² and 13.52 kg/mm² were used. This was not felt to be an objection, rather the contrary; since the purpose of these tests was to show the effect of rather rare very high loads these loads were chosen higher than necessary for a program-fatigue test. In fact the limitation of the high loads was the allowable static overloading of the dynamometer of the fatigue machine. The gust spectrum of Taylor used in this test program was published in 1953 and since then newer data became available. For the more up to date gust spectrum, published by Bullen (ref. 4), the slope of the spectrum is somewhat steeper. This means that the higher gust velocities are relatively slightly more frequent than in Taylor's spectrum and the low gust velocities are relatively slightly less frequent. On the basis of Bullen's data it could be shown that the frequency of the periodic high loads was of the correct order of magnitude relative to the frequency of the cycles at the level of maximum damage.

5. The results of the program-fatigue tests.

The results of the individual tests are presented in tables 5 and 6. The endurance of each test is given in three different ways, viz.

- (1) the endurance in load cycles (N');
- (2) the endurance in number of periods and
- (3) the endurance expressed as $\sum \frac{n}{N}$.

Since all specimens of one test series were subjected to the same load spectrum, they were loaded at a slightly different stress spectrum due to a variation in sheet thickness for different specimens. For the calculation of $\sum \frac{n}{N}$ the N -values corresponding to the actually applied S_a -values have been used.

Since endurances are almost always averaged on a logarithmic basis this procedure has been adopted for $\sum \frac{n}{N}$ as well. A numerical indication of the scatter of the endurance was obtained by calculating the standard deviation and

the coefficient of variation of $\log N'$. The mean results expressed as $\sum \frac{n}{N}$ are also given in figs. 14 to 17 and the scatter has been compared with the scatter of the constant-amplitude tests in fig. 18.

6. Review of the results of program-fatigue tests of other investigations.

6.1. Scope of the chapter

The results of other investigations will be presented in this chapter. A general discussion of these results and a comparison with the present investigation will be given in the following chapter.

The review will be restricted to program-fatigue tests on notched specimens or structures, manufactured from light alloys. In a previous report (ref. 45) the authors made a similar review including unnotched specimens. There it became clear that unnotched specimens may exhibit a cumulative-damage behaviour which is quite different from the behaviour of notched specimens. For that reason unnotched specimens are now omitted from the review. The previous review was prepared in 1955. Another review has been compiled by Weibull (ref. 58) in 1956. Since then many new data became available. Additional information on unnotched specimens, not discussed in ref. 45 or this report, may be found in refs. 14, 15, 23, 26 and 33.

It has been tried to summarize the most important results of other investigations in the text of this chapter. More details are contained in figs. 19 to 47, to which references are made in the text, but the reader need not consult these figures if he only desires to obtain a general review. In the figs. 19 up to 47 it has been tried to present for each investigation the most characteristic data, such as: type of specimen, stress concentration factor, material, load spectrum, load sequence, maximum and minimum stress level, the number of tests, etc. The severity of the maximum and minimum stress levels has been further indicated by the corresponding endurances. All these data were not always available.

6.2 Results of investigations on small specimens.

The first program-fatigue tests were performed by Gassner (ref. 16). A review of his extensive results is given in ref. 18. The aim of Gassner's work was not to check the Palmgren-Miner rule, which probably at that time, about 20 years after Palmgren's publication (ref. 35), was not very well known. The purpose of

Gassner's investigations was to show that the airworthiness requirements for static strength were not sufficient with respect to fatigue and to present program-fatigue tests as a more reliable basis for a fatigue evaluation. For these purposes he tested notched specimens from a number of aircraft materials with two different types of load spectra, viz. a gust spectrum and a combined gust and maneuver spectrum. Moreover, Gassner realized that a program-fatigue test can be performed in many different ways and he studied the parameters involved. Since the relevant constant-amplitude fatigue data were available (fig. 19), the authors have calculated the $\sum \frac{n}{N}$ -values for Gassner's tests.

A survey of the results is presented in figs. 19 to 22. It should be pointed out that the accuracy of the calculated $\sum \frac{n}{N}$ -values is not very high, due to the fact that the N-values had to be obtained from a Goodman-diagram determined by a number of specimens which was fairly small for this purpose. Further the results presented are individual test results. However, since the number of tests is large some trends can be derived from them. It may be said that on the average the results show that $\sum \frac{n}{N} \geq 1$. Fig. 19 shows the results with the gust spectrum and fig. 20 shows the results with the combined gust and maneuver spectrum. Gassner also studied the effect of omitting the lowest stress amplitude, see fig. 19, which produced higher $\sum \frac{n}{N}$ -values. Figs. 21 and 22 give an indication of the effect of varying the length of the period and employing different load sequences. It seems that the effect of both parameters on $\sum \frac{n}{N}$ is not very large. Finally Gassner studied the effect of the number of stress levels by approximating the continuous gust spectrum with 2, 3, 4 and 9 discrete levels. The results were presented by Gassner as the maximum allowable stress for a total life of 5,400,000 cycles. An evaluation to $\sum \frac{n}{N}$ -values was not possible since the number of cycles per period for each level was not known exactly, except for the tests with 9 stress levels. Since the differences in allowable maximum stresses for 3, 4 and 9 levels were small the conclusion is allowed that the differences in $\sum \frac{n}{N}$ -values would have been small too.

More recently (ref. 21) Gassner published results of program-fatigue tests on rotating bending specimens ($S_m=0$) and two types of spectra, see fig. 23. For the gust spectrum $\sum \frac{n}{N}$ -values are slightly below one and for the maneuver spectrum slightly higher than one. More tests on this type of specimen are in progress.

Extensive results have been published by Wallgren and co-workers, see figs. 24 to 27. Fig. 24 (ref. 53) presents the most extensive re-

sults for different specimens, 2 load spectra and 2 materials. For almost all tests $\sum \frac{n}{N} > 1$. Wallgren also studied the effect of omitting the stress levels below the endurance limit. From fig. 24 it can be seen that this omission has a slight beneficial effect. Fig. 25 gives results from Wallgren and Petrelius (ref. 54) on a bolted joint with a maneuver spectrum. $\sum \frac{n}{N}$ was noticeably higher than one. Fig. 26 gives results reported by Wallgren (ref. 55) on a simply notched bar. Sufficient tests were performed to allow an evaluation on a probability basis. For the probabilities of failure for which $\sum \frac{n}{N}$ has been calculated, it was clearly beyond one. In fig. 27 the results of Wallgren and Svensson (ref. 56) on simple lugs with a maneuver spectrum are presented for two light alloys. $\sum \frac{n}{N}$ -values are again higher than 1.

Barrois (ref. 2) has reported results of tests in which the effect of the "ground-to-air cycle" has been studied. These cycles have been inserted at regular periods between an almost constant amplitude loading, see fig. 28. The period was studied as a variable. The specimen was a riveted joint. The results show that for such a loading, which is almost similar to the loading in the RAE-test, $\sum \frac{n}{N}$ -values are systematically below unity and of the order of 0.5.

Fisher (refs. 10 and 11) performed tests on a specimen with two side notches. He employed a maneuver spectrum as well as a gust spectrum, see figs. 29 and 30. For both spectra $\sum \frac{n}{N}$ -values were slightly above one. For the gust spectrum Fisher studied the effect of halving the period length and the effect of omitting the lowest S_a -values. The former had almost no effect and the latter had a small beneficial effect.

Smith (ref. 49) studied the cumulative damage behaviour of a specimen with a central hole under a maneuver spectrum, see fig. 31. Based on the normal S-N-curve he found $\sum \frac{n}{N} = 4.9$. However, based on the S-N-curve for specimens pre-loaded to the highest stress occurring during the program-fatigue test this value reduced to $\sum \frac{n}{N} = 1.65$.

Extensive test series have been carried out by Hardrath, Utley and Guthrie (ref. 26) on rotating beam specimens and by Naumann, Hardrath and Guthrie (ref. 35) an axially loaded sheet specimens. A surveying paper has been presented by Hardrath and Naumann (ref. 25). The results are summarized in fig. 32 to 36, most figures being derived from ref. 25.

The notched rotating-beam specimens have been tested with three different load spectra, viz. a sinusoidal spectrum, an exponential spectrum and a gust spectrum. The results

for the latter are shown in fig. 32. The results for the other spectra are not given because they showed a fairly large scatter. Qualitatively they showed the same trend as the gust spectra, i. e. $\sum \frac{n}{N} < 1$. The results varied from 0.25 to 0.86 with an average of 0.49. For the gust-spectrum tests the mean value was 0.42.

The axial-load tests are of much more importance. The aims of the investigation were to a large degree identical to those of the present investigation. The parameters studied were: type of load sequence, period length, mean stress, type of material and number of stress levels. Fig. 33 shows the effect of different load sequences the highest results being obtained with decreasing stress levels and the lowest results with increasing stress levels. This is the opposite effect as found in the present investigation.

Fig. 34 shows the effect of the mean stress. The trend is obvious. Higher mean stresses lead to higher $\sum \frac{n}{N}$ -values. It should be noted that a variation of S_m was accompanied by a variation of the S_a -spectrum. The maximum stress remained the same and was about equal to the stress at the design limit load.

The effect of the length of the period is illustrated in fig. 35. The effect seems to be small for the 7075-specimens. The 2024-specimens reveal a less systematic influence. Lower $\sum \frac{n}{N}$ -values seem to be associated with the higher period length.

In fig. 36 a comparison is made between the results for 2024 and 7075 material, the latter material consistently giving the higher $\sum \frac{n}{N}$ -values.

Naumann, Hardrath and Guthrie performed two test series on 2024 specimens with the same spectrum, the same number of cycles per period and the same load sequence (randomized sequence), but with a different number of stress levels, viz. 8 levels in one test series (4 specimens) and 18 levels in the other test series (also 4 specimens). The results were respectively $\sum \frac{n}{N} = 1.02$ and $\sum \frac{n}{N} = 1.16$, showing no significant difference.

6.3. Results of investigations on structures.

Figs. 37 and 38 summarize the results of the random and program-fatigue tests on Mustang wings, reported by Payne (refs. 37 and 38). In the random-fatigue tests each load amplitude and its sign were selected at random. This led to a rather irregular load sequence as illustrated in fig. 37. The load spectrum used in these tests was essentially a gust spectrum, however, it is a rather severe spectrum since a 50 ft/sec gust involves a maximum load of 92% P_u . This was purposely done so to avoid very long endurance in the random tests.

Also the landing cycle (ground-to-air cycle) is rather severe (-24.2% P_u). It can be seen from the second column of the second table in fig. 37 that these landing cycles cut down the endurance with a factor on life of about 5. Calculation of $\sum \frac{n}{N}$, disregarding the type and the location of the failure were made according to 4 different hypotheses, outlined in fig. 37. Hypotheses H_1 and H_2 correspond to the peak-method and the range-method which will be discussed in section 7.5d. H_2 gave less conservative results than H_1 . Unconservative results were also obtained if the damage of the landing cycles was calculated separately (hypothesis H_3). Hypothesis H_4 is identical to H_1 except that now N -values are used obtained from specimens, preloaded until the highest load occurring in the test. Results obtained with H_1 and H_4 were not very much different. It should be kept in mind that the comparison of these four hypotheses is based on results which do not take into account the type and location of failure.

The most interesting data for checking the Palmgren-Miner rule are those for which the type and the location of failure were accounted for. Unfortunately these data are the least reliable since life estimates based on crack growth data had to be made in all those cases for which crack growth of different failure locations overlapped and not all cracks could be followed until final failure. From the results in the right half of the second table in fig. 37, evaluated according to hypothesis H_1 , it turns out that now these results on the average are on the safe side.

According to Payne (ref. 38) predictions for the crack rate in the gun bay area were made for the random fatigue tests without ground-to-air cycles with the crack rate data of the constant amplitude tests as a basis. It then turned out that hypothesis H_2 (range-method) gave a better representation than H_1 , the latter being too conservative. This is confirmed by $\sum \frac{n}{N} = 2.66$ for the final failure in this area with H_1 , since for this load sequence and this failure location almost the complete life was covered by crack propagation.

Fig. 38 gives the results for the program-fatigue tests on the Mustang wings. Three load levels were used. It was suggested at that time by the Australian airworthiness authorities that such a type of test might give valuable indications of the airworthiness of a structure with respect to fatigue. Again the results have been evaluated in two ways, i. e. disregarding and considering the type of failure. For the latter case, which is the more interesting one, $\sum \frac{n}{N}$ -values are all above unity.

Carl and Wegeng (ref. 5) have tested outerwing panels of a jet-fighter with a maneuver spectrum, see fig. 39. All three load levels of the program were high and corresponded to low endurance. $\sum \frac{n}{N}$ -results are of the order of 1.5.

Fig. 40 summarizes the results of so-called randomized-step tests on Commando wings, reported by Whaley in ref. 59. A randomized-step test is a special type of program-fatigue test. Within one period all load cycles of the same magnitude are applied in one batch. The batches are arranged in a random order by consulting a table of random numbers. Therefore, the degree of randomness is much lower than for the tests on the Mustang wings of fig. 37. The interpretation of the test results met with the same difficulties as for the Mustang wings, i.e. there was more than one failure location and not all types of failure occurred in all tests. Disregarding the type of failure the $\sum \frac{1}{N}$ -values were considerably higher than unity.

The results of fig. 40 were based on tests on 3 wings (6 outer wing panels). According to Huston (ref. 29) more panels are tested at the moment, also with a maneuver spectrum. Quantitative data on these additional tests have not yet been published. However, Huston gives some data on a very urgent aspect of the problem, which is the relation between the endurance in laboratory tests and the actual endurance under service conditions. These data are summarized in fig. 41. Apparently the agreement between both types of data is reasonably good. Nevertheless Huston carefully avoids this quantitative conclusion. Indeed one should be very careful to draw this conclusion with its far reaching implications on the basis of the results summarized in fig. 40 and 41, because :

- (1) There are uncertainties about the load history of the aircraft in service.
- (2) The scatter in the laboratory tests for the time until crack initiation was fairly large.
- (3) In these tests 8 different types of cracks were found in a limited area of the structure just beyond the joints of the center section to the outer panels. This part of the structure contained many cut outs and from the point of view of fatigue it was probably not an ideal structure.

The results in fig. 41 are still presented, despite their limited quantitative value, because they illustrate the type of data which form a very important missing link in the whole problem of aircraft fatigue. It may be hoped that more data on the comparison between laboratory tests and service experience will become available.

One other reference could be found in the literature concerning the comparison of service experience and laboratory test results. In the Aircraft Fatigue Handbook of the Aircraft Industries Association (ref. 60) the fatigue life is calculated for a wing skin joint, see fig. 42. This calculation is based on the relevant S-N-curve with an estimated loading in service. The calculated fatigue life, 7900 flying hours, compares favourably with the fatigue life in service, which was 10,000 to 12,000 flying hours.

6.4. Results of tests with very high loads.

The results of tests with high loads and pre-loads as reported by Heywood (ref. 28) have been presented in figs. 43, 44 and 45. The tests were constant-amplitude tests, preceded by a high pre-load or to which periodical high loads were inserted. Fig. 43 shows that a single pre-load has to be fairly high to be effective. Fig. 44 shows the effect of periodical high positive loads. The curve for one high pre-load is shown in this figure as a reference. It turns out that periodically applied high loads are far more effective than a single pre-load. In this figure there are also four results for a batch of 10 pre-loads. Their effect does not seem to be much different from one pre-load, but the number of tests is too small to allow a conclusive statement.

For comparison with the present investigation the most interesting data of Heywood are given in fig. 45, since this figure shows that the beneficial effect of periodic high positive loads is largely eliminated if these loads are immediately followed by negative loads.

In spite of the scanty number of tests the results are in full agreement with the present investigation.

Results of a high pre-load are also presented by Payne (ref. 38) for the Mustang wings. They are in agreement with Heywood's results and confirm that very high pre-loads are necessary to obtain a beneficial effect.

6.5. Results of random-noise-fatigue tests.

In three investigations (refs. 12, 26a and 30) random noise has been used as an input to an electromagnetic vibrator loading small notched specimens in plane bending. A survey is given in fig. 46. For all three investigations the relevant S-N-curves were determined, thus allowing an evaluation of $\sum \frac{1}{N}$.

Kowalewski (ref. 30) also performed program-fatigue tests with the same test set-up, employing a program which was derived from the random stress variations, see fig. 47. This figure is further discussed in section 7.5d.

All three investigations were conducted with $S_m = 0$. The stress concentration factor was very low for the investigation of Head and Hooke, moderate for the investigation of Kowalewski and high for the investigation of Fralich. This is clearly reflected in the values of the fatigue limits. There is a correspondingly large divergence in $\sum \frac{1}{N}$ -values. Fralich reported that a large part of the fatigue life of his specimens was covered by crack propagation, which is usual for severely notched specimens (ref. 46). The typical properties from random noise as a load-time history will not be discussed here. Some remarks are presented in section 7.5d.

7. Discussion .

7.1. Scope of the chapter .

In this chapter the results of the present investigation will be discussed and compared with the results of other investigations reviewed in the previous chapter. In view of the many aspects of the problem it has been tried to establish a systematic sub-division for this chapter with the objectives of this investigation, outlined in the introduction, as a guide.

The main questions are:

- (a) How is the fatigue life in a program-fatigue test affected by the parameters of the program, such as period length, load sequence, highest and lowest load level to be included, etc?
- (b) What can be said about the validity of the Palmgren-Miner value under program-fatigue loading? What is its value for design purposes?
- (c) What is the value of program-fatigue loading for design purposes?

Topic (a) is dealt with in section 7.2, (b) in 7.4 and 7.6 and (c) in 7.5 and 7.7. As special topics the effect of periodic high loads and scatter in program-fatigue testing are discussed in sections 7.3 and 7.8 respectively.

7.2. The effect of different parameters of the program-fatigue test on the endurance.

7.2a. The load sequence in a period.

Sequences to be discussed are: (1) increasing stress, (2) decreasing stress and (3) increasing and decreasing stress. These sequences are illustrated in fig. 15. Random and randomized sequences are discussed in another part of this chapter.

The results of the present investigation are summarized in fig. 15. The effect of the load sequence is small for the 7075 specimens. For the 2024 specimens, however, there is a clearly noticeable effect, the smallest endurance being obtained by the decreasing sequence and the highest endurance by the increasing sequence. It is believed that the explanation must be based on favourable internal stresses formed at the highest stress level.

For the increasing sequence the highest stress level is immediately followed by the lowest stress level and due to the favourable internal stresses the cycles at the lower stress levels will be more or less ineffective.

For the decreasing order of stress levels the lowest stress amplitude will have a better chance to be damaging since then the preceding stress levels may have induced some relaxation of the internal stress system. The relaxation may be due to plastic deformation in the

critical region, but also to continued crack propagation.

It is not surprising that this effect is only found for 2024 material, since the maximum stress for specimens of this material was 17 kg/mm² (24.2 k.s.i.), compared with 12.6 kg/mm² (17.9 k.s.i.) for 7075 specimens. Moreover the yield stress is markedly lower for 2024 than for 7075.

Additional evidence for the explanation given above is offered in fig. 17, compare test series 6 and 17. The beneficial effect of periodic positive high loads is much higher for an increasing load program than for a decreasing one.

Similar tests were performed by Gassner (ref. 18), see fig. 22 and by Hardrath et al. (refs. 25 and 35), see fig. 33. Gassner found a small effect in agreement with the results of the present investigation for 2024-specimens. Hardrath found a large effect of the sequence for 2024 and 7075 specimens both. Strangely enough, this effect was opposite to the effect as found in the present investigation and by Gassner. No explanation can be offered. In agreement with the present investigation the results for the tests with increasing and decreasing stress levels succeeding each other were intermediate to the results for increasing levels only and decreasing levels only.

7.2b. The length of the period.

The effect of the number of cycles in one period as found in the present investigation, is shown in fig. 16. Tests were performed on 7075 material only. The ratio's of the period lengths are 4:2:1. A slight effect on the endurance is found, the smallest endurance being associated with a small number of periods. The highest endurance was found for 12 periods covering the complete life. It is noteworthy that Gassner (ref. 18) found exactly the same trend, see fig. 21. Fisher (ref. 11) studied 2 period lengths with a ratio of 1:2 and he found a very small effect, see fig. 30. In fig. 35 results of Hardrath et al. (refs. 25 and 35) are shown. These results on the average confirm the view that no large effects are found. However, for the 2024 material there is an indication that long periods (a small number of periods until failure) might give lower ΣN -values. This may be due to applying the cycles with the lower stress amplitudes in relatively large batches, thus increasing their possibility to be damaging. It is then felt that long periods are an unrealistic approach and it may be concluded that the effect of the period length, although not fully negligible, will be small provided that a small number of periods, until failure, say smaller than 10, is avoided.

7.2c. The lowest S_a to be included in the program.

In reducing a load spectrum to a program for a program-fatigue test it is a problem to set a lower limit to the S_a -values which should be included in the test. Including cycles with very low S_a -values implies large testing times because for practical spectra these cycles are very numerous. According to the Palmgren-Miner rule it should not be necessary to include cycles whose stress amplitudes are below the endurance limit since such cycles do not contribute to $\sum \frac{a}{N}$ because $N = \infty$. This point is checked in some test series of the present program, see fig. 14. Comparison of test series 10, 11 and 15 for 7075 specimens and test series 21 and 25 for 2024 specimens reveals a consistent tendency to lower $\sum \frac{a}{N}$ -values by adding load cycles with lower S_a -values to the program. This effect may be explained by assuming that load cycles with a low S_a which are unable to create fatigue cracks if applied separately will be able to extend existing cracks created at higher stress levels in a program-fatigue test.

The results of the present investigation are confirmed by those of Gassner (ref. 18), Wallgren (ref. 53) and Fisher (ref. 11), see figures 19 (footnote 3), 24 (footnote) and 30 (program 2 and 3) respectively. The increase of the $\sum \frac{a}{N}$ -values by omitting cycles below the endurance limit amounts to about 50% on the average. Now it is felt that a program-fatigue test gives cycles with low stress amplitudes the maximum chance to be damaging, because in such a test these cycles are applied in batches, each consisting of a large number of cycles without intermediate higher stress cycles. This is felt to be an unfavourable sequence which does not occur in the more random sequences which will be encountered in service. It may be concluded that it will be a safe procedure to include load cycles with a stress amplitude below the endurance limit, but it seems to be an unnecessary penalty to include stress cycles far below the endurance limit. To give a more quantitative advice on a rational basis is impossible. Other aspects which are of importance in this respect are:

- (a) the quality of the estimation of the fatigue limit and
- (b) the practical interpretation which has to be given to the result of the program-fatigue test.

In performing program-fatigue tests on large structures load cycles with a low amplitude should be included as far as practicable since these load cycles may produce a mode of failure which otherwise might not be found, for instance by crack initiation due to fretting.

7.2d. The highest S_a to be included in the program.

Another problem in deriving a load program is the establishment of the upper limit to the stress amplitudes to be included.

Results of the present investigation are shown in fig. 14. Comparison of test series 10, 3 and 2 for the 7075 specimens and test series 21 and 24 reveals that omitting the highest S_a leads to lower $\sum \frac{a}{N}$ -values. Comparison of test series 18 and 1, also in fig. 14, does not show any effect. More or less similar tests have been performed by Gassner (ref. 18) and Wallgren (ref. 53), see figs. 19, 20 and 24 resp. Results of these investigators are in agreement with the present investigation.

They show that lower values of the maximum applied stress in the program (S_{max}) lead to lower $\sum \frac{a}{N}$ -values (not necessarily below unity). Increasing S_{max} leads to higher $\sum \frac{a}{N}$ -values until a maximum is reached. For very high values of S_{max} again a decrease of $\sum \frac{a}{N}$ values is noticed. Maximum values of $\sum \frac{a}{N}$ seem to be promoted by values of S_{max} corresponding to an endurance of about 10000 cycles.

Gassner's results are somewhat less systematic, which is not strange, since fig. 19 represents individual test results.

The fairly low S_{max} -value for the 7075 specimens of the present investigation may account for the low $\sum \frac{a}{N}$ -values found in fig. 14.

An explanation seems to be possible. Cycles with a high S_a induce two counteracting effects, viz. a favourable effect which is the formation of residual stresses and an unfavourable effect which is the relatively large contribution to the crack growth. In general the first effect will be the dominating one; however, it will be clear that this effect cannot be increased indefinitely and therefore the second effect may become dominating at very high values of S_{max} .

It is felt that Gassner's results on rotating beam specimens, see fig. 23, are consistent with the explanation. For these tests $S_m = 0$ and the formation of favourable residual stresses will be more difficult. Consequently no increase of $\sum \frac{a}{N}$ with increasing S_{max} and no such maximum for $\sum \frac{a}{N}$ are found.

In view of the effect of S_{max} on the fatigue life in program-fatigue tests the most acceptable approach to establishing a program for a test on a structure or part of it, seems to be to select that level that may be expected to occur a not too small number of times in the anticipated life of most aircraft, for instance 10 to 100 times. The effect of higher loads is discussed in section 7.3.

7.2e. The number of load levels.

Once the minimum and maximum amplitude of the program have been settled one has to decide

how many intermediate stress levels will be employed. Systematical investigations were performed by Gassner (ref. 18) and Naumann et al. (ref. 35). In this respect both investigations were of a limited extent. Gassner did not find much difference in fatigue life by approximating the spectrum by 3, 4 or 9 stress levels, S_{amin} and S_{amax} being the same in all cases. Also Naumann et al. testing the fatigue specimens from fig. 33 and using a gust spectrum, a randomized sequence with 8 or 18 stress levels, found only a small difference. Despite the scanty evidence it is believed that the effect of the number of stress levels will be small if this number is not too small. From microscopical studies it is known that a difference can be made between high and low-level fatigue. However, it is not sure that this difference is essential rather than gradual. It is thought that the phenomenological knowledge of fatigue does not necessitate a very high number of stress levels, although it will never be an objection. A minimum number of 6 to 8 may be acceptable.

7.3. Periodic high loads and ground-to-air cycles.

From the results presented by Heywood (ref. 28) and reproduced here in figs. 43 and 44, it was known that a high preload may significantly increase the fatigue life under constant amplitude loading if the preload is very high, i. e. of the order of the limit load or higher. Tests on Mustang wings, reported by Payne (ref. 38) have confirmed this. Further it was shown by Heywood that high loads inserted periodically in a constant amplitude test increase the fatigue life much more than one preload of the same magnitude. Heywood advanced two explanations. (1) The internal stresses may decrease during fatigue testing and they are restored again by the periodic high loads. (2) Microcracks or cracks may have formed. Their growth will be retarded by the internal stresses built up around these cracks by the periodic high loads. Both reasons may apply. Heywood's results apply to constant-amplitude fatigue tests. In the present investigation the effect on the fatigue life under program-fatigue loading was studied. From the results, see fig. 17, the following conclusions may be drawn. One high preload increased the fatigue life only to a limited extent. However, periodic high positive loads, though smaller than the preload, increase the life considerably, compare test series 10, 5 and 6. Comparison of test series 6, 6a and 6b shows that stopping the application of the periodic high loads leads to the end of the fatigue life after an additional $\sum \frac{n}{N}$ -increment of about 1 and that lowering the periodicity of the high loads about twice (corresponding to a period of $\sum \frac{n}{N} \sim 0.2$) is also accompanied by a decrease in fatigue life.

The results are in perfect agreement with Heywood's results and his explanation. An interesting comparison is possible between test series 6 and 17, the first test series being conducted with an increasing load sequence and the second test series with a decreasing load sequence. There is an appreciable difference in fatigue life consistent with the trend discussed in section 7.2a. Also here it is felt that the decreasing load sequence will give a more rapid destruction of the favourable internal stress system than the increasing load sequence, it may be either by crack propagation or by stress relaxation.

A comparison of test series 10, 6 and 7 reveals that periodic high negative loads are harmful, but the unfavourable effect is less than the favourable effect of periodic high positive loads, although in test series 7 the amplitude of high negative loads ($20\% + 39\% S_U$) was larger than for the high positive loads ($39\% S_U$) in test series 6. Two additive explanations are valid:

- (1) For riveted and bolted joints the critical section is less highly stressed in compression than in tension and
- (2) if fatigue cracks exist, they may be closed under compression loads, thus diminishing the stress concentration.

Finally test series were performed with periodic high load cycles consisting of one high positive load followed by one high negative load or in the reversed order. Comparing test series 8 and 9 for 7075 specimens and test series 27 and 28 for 2024 specimens it will be clear that from such a high load cycle the last half cycle has the dominating influence. This is confirmed by one test result of Heywood presented in fig. 45.

Probably the main result of these tests with periodic high loads is an illustration of the effect of residual stresses. The results emphasize the problem that if such rather rare very high load cycles have to be inserted in a program-fatigue test one has to consider the question if the positive or the negative half cycle has to be applied first. Now, it should be realized that the present test series have some extraordinary features. Firstly, the difference between the high load cycles and the highest load cycles of the basic program is too large to consider the high load cycles as a normal part of the program, and, secondly, the high load cycles have been added almost exclusively to a load sequence with increasing stress levels only. It is felt that under more normal testing conditions the effect of the highest load cycle occurring once in a period would have been less. Nevertheless, as a conservative measure, it is advised for practical program-fatigue testing that high loads occurring with a frequency of less than say ten in the anticipated fatigue life (on a basis of $\sum \frac{n}{N} = 1$) should not be included.

There is one ever-recurring type of load for an aircraft, which is the so-called ground-to-air cycle or landing cycle. For many parts of an aircraft this ground-to-air cycle implies a negative load deviation from the loading in flight. From the results of test series 7 with the periodic high negative loads one might assume that the ground-to-air cycle may have a considerable damaging effect. Now in test series 7 the negative loads were too high and too less frequent to represent the ground-to-air cycles. This topic has been especially studied by Barrois (ref. 2). His results are presented in fig. 28. From his results, the average being $\sum \frac{n}{N} = 0.5$, it is clear that the ground-to-air cycles cause considerably more damage than according to the Palmgren-Miner rule. This is confirmed by the results on the Mustang wings reported by Payne (ref. 38), see fig. 37. For the Mustang wings the ground-to-air cycle was rather severe. They decreased the life with a factor of about 5. No calculated values of $\sum \frac{n}{N}$ were available for the Mustang wings, which account for the location of failure and for which the damage of the ground-to-air cycle has been calculated separately (hypothesis H_3). Probably $\sum \frac{n}{N}$ -values below unity would then have been obtained. The procedure of Payne, to treat the ground-to-air cycle as a gust loading (hypothesis H_1), which implies an asymmetric gust spectrum, seems therefore to be advisable for life calculations. More experimental research concerning the effect of the ground-to-air cycle is certainly desirable. Until then one should keep in mind the unfavourable effect of this type of loading, which may well be due to the periodical destruction of any favourable internal stress system. * It appears to be essential to include this type of loading in program-fatigue test on structures.

* After this report had been completed the results of a systematical study on the effect of ground-to-air cycles was published by Gassner and Horstman (E. Gassner and K. F. Horstmann: Einfluss des Start-Lande Lastwechsels auf die Lebensdauer der böenbeanspruchten Flügel von Verkehrsflugzeugen, Lecture presented at the 2nd ICAS-Congress, Zürich, sept. 1960). This investigation offers a substantial confirmation of the discussion presented here and conclusion 8 of chapter 8.

7.4. The validity of the Palmgren-Miner rule.

In fig. 50 a survey is presented of the investigations discussed in the previous sections. There is a fairly large variation in $\sum \frac{n}{N}$ -values and this will now be discussed for small specimens and structures separately. The validity of the rule will be discussed irrespective of the question whether program-fatigue testing is a correct representation of service loads. This problem is discussed in section 7.5. The value of the Palmgren-Miner rule for design purposes will be discussed in section 7.6.

7.4a. Small specimens.

In general small specimens show only one mode of failure and crack propagation such as it occurs in structures is impossible. However, it is felt that the fatigue life of a small specimen may be compared with the fatigue life of a structure until the time that a "technically visible crack" has been initiated. A technically visible crack is a concept which does not allow a concrete definition, however, one may think of a crack length in the order of 1 cm ($\sim 3/8''$). Such cracks may be found by the usual procedures for service inspections. The comparison seems to be allowed since that part of the fatigue life of a small specimen during which the stress due to a certain load is seriously affected by crack propagation is fairly small.

Also for small specimens alone fig. 50 shows a large variability of $\sum \frac{n}{N}$ -values. However, there are some more or less systematical trends to be observed.

Low values of $\sum \frac{n}{N}$, i.e. values below one, will first be discussed. These values imply unconservative life estimates if the Palmgren-Miner rule is accepted. From fig. 50 it may be deduced that low values of $\sum \frac{n}{N}$ are obtained in the following cases:

- (a) Rotating beam specimens tested by Hardrath et al. $S_m = 0$.
- (b) Random noise tests by Head and Hooke. $S_m = 0$, $k_t = 1.33$
- (c) Some tests by Gassner.
- (d) Tests by Barrois with ground-to-air cycles.
- (e) Tests by Naumann et al. $S_m = 0$.

From this list it is clear that $S_m = 0$ in general leads to low $\sum \frac{n}{N}$ -values. This is consistent with the view that for $S_m > 0$ the possibility for the formation of favourable residual stresses is much better than for $S_m = 0$. With respect to the results of Gassner it should be remembered that his results for plane bending and axial loading were individual test results. So Gassner's lowest results cannot be regarded

as indications of average trends. Barrois found $\sum \frac{n}{N} \sim 0.5$ if ground to air cycles were superimposed on a constant amplitude loading. It may be concluded that unconservative life estimates with the Palmgren-Miner rule may be obtained if $S_m = 0$ or if ground-to-air cycles are important. In addition it has to be said that ground-to-air cycles do not necessarily involve that $\sum \frac{n}{N} = 1$ is unsafe. Without ground-to-air cycles Barrois would obviously have obtained $\sum \frac{n}{N} = 1$ (constant amplitude loading). Since program-loading with $S_m > 0$ may easily result in $\sum \frac{n}{N} > 1$, the addition of ground-to-air cycles to such a loading may still yield a result of $\sum \frac{n}{N} > 1$ or even higher. This is a problem which should be further investigated.

High values of $\sum \frac{n}{N}$, i. e. values higher than one, are according to fig. 50 not at all seldom. On the contrary, such values are dominating. It has been pointed out in section 7.2d that high $\sum \frac{n}{N}$ -values may be obtained if load cycles with fairly high stress amplitudes are included in the test. Very high values of S_a may again introduce some lowering of $\sum \frac{n}{N}$. Average values for axial load tests with $S_m > 0$, excluding the results of Gassner (individual test results) and Barrois (ground-to-air cycles) are of the order of 1.5 to 2.0. The results of the present investigation for 7075 specimens seem to be an exception, $\sum \frac{n}{N} = 0.6-1.1$; however, it is felt that this will have been due to the fairly low maximum stress applied in these tests. It is concluded that for program-fatigue loading without ground-to-air cycles the Palmgren-Miner rule will underestimate the fatigue life.

From the survey of results in fig. 50 it may be concluded that there is little difference in the cumulative-damage behaviour of 2024 specimens and 7075 specimens. On the average the 7075 specimens do not produce lower $\sum \frac{n}{N}$ -values. This does not imply that both materials are equivalent from the fatigue point of view. Since S-N-curves for both materials in the notched condition are generally equivalent or worse for the 7075 specimens, the fatigue life expressed in flying hours will be lower for the 7075 specimens than for 2024 specimens of the same geometry and designed to the same load factor.

From fig. 50 it will be seen that the majority of load programs was associated with a gust spectrum or a maneuver spectrum. No systematical difference in the results for both spectra is apparent. It is thought therefore that for load spectra intermediate between a gust and a maneuver spectrum similar results may be expected.

Obviously the unfavourable effect of the ground-to-air cycle will be negligible for a maneuver spectrum loading if negative maneuvers with

the magnitude of the ground-to-air cycle are not seldom.

7.4b Structures.

For obvious reasons the number of investigations on structures is much smaller than on specimens. Moreover the information is obscured by the interference of different types of failure. The general impression is that for the fatigue life including the crack propagation in the structure the Palmgren-Miner rule will certainly be on the safe side. The Australian results on Mustang wings indicate that for a gust-spectrum loading the ground-to-air cycle may be a severely damaging load. The damage of these cycles should not be calculated separately, but the ground-to-air cycles should be treated as being negative gust loads, thus involving an asymmetric load spectrum. Under such conditions Payne, employing the peak-method, see chapter 7.5, obtained $\sum \frac{n}{N} \sim 1$ for final failure.

For the time until visible cracks in a structure have grown, the conclusions of the previous section are supposed to be valid as explained there. The crack propagation in a structure has to be considered separately. However, hardly any data are available in this respect. The impression obtained from the tests on the Mustang wings is certainly that for the crack-propagation stage in a structure the Palmgren-Miner rule will be on the safe side. This is not strange since cracks allow the formation of even more intensive residual stress fields than notches. It has been observed (refs. 24 and 7) that changing the load amplitude during fatigue crack propagation yields an interaction from one load level of the crack propagation at the subsequent load level. It is strongly advised to study crack propagation in structures or components under program-fatigue testing, both with and without ground-to-air cycles.

7.5. Differences between program-fatigue testing and service loading.

For a program-fatigue test on a full-size structural component or structure, there are a number of obvious differences with the loading of an aircraft under service conditions. If program-fatigue loading has to represent loading in service it is important to know whether these different circumstances may have a noticeable influence on the fatigue life. In this section the following aspects will be discussed.

- (1) The rate of loading
- (2) Rest periods
- (3) The environmental conditions
- (4) The load sequence for program-fatigue loading vs. random service loading.

7.5a. The effect of the rate of loading.

The effect of the frequency of cyclic loading on the fatigue life has been reviewed recently by Stephenson (ref. 50). The majority of investigations has been performed on unnotched specimens. In the technically important range from 10 to 10^4 cycles per minute (sonic fatigue and fuselage-pressurisation cycles being left out of consideration) the effect seems to be fairly small for light alloys at room temperature. If any effect is present lower frequencies give lower endurance and the effect will be most noticeable at higher stress amplitudes. More empirical research for notched specimens seems to be advisable.

At the moment the frequency effect on the crack rate in a sheet is studied at the NLL. Preliminary results show a 30% higher crack rate at a frequency of about 20 cycles per minute as compared with 2000 cycles per minute. It is thought to be possible that a frequency-dependent creep effect will occur around the tip of the crack. From this point of view a smaller frequency effect may be expected for the time until crack initiation.

It is regrettable that the physical concepts of fatigue do not yet offer much guidance with respect to the frequency effect. No speculation will be presented here.

7.5b. The effect of rest periods.

An aircraft is not continuously flying and the structure has ample time to relax if such a mechanism is possible. Some investigators have therefore studied the effect of rest periods on the fatigue life. Gassner (ref. 17) has employed program-fatigue loading for this purpose. His results are summarized in fig. 48 and some results are contained in fig. 19 (see footnote 4.) The results are fairly scanty and show perhaps a slightly beneficial effect of the rest periods.

Constant amplitude tests with and without rest periods were conducted by Bollenrath and Cornelius (ref. 3) and by Gunn (ref. 23) and they did not indicate any effect. Smith (ref. 49) has

studied the effect of one high pre-load on the endurance of a notched specimen. His results are shown in fig. 49. It is clear that a relaxation period of half a year did not at all impair the beneficial effect of the high preload. Also here it is difficult to employ physical arguments. Diffusion and some dislocation movements may probably go on during rest periods, but even a prediction whether this would be beneficial or harmful in an unstable light alloy is extremely speculative. The view that diffusion and dislocation movements during rest periods will occur on a much smaller scale than during alternated stressing is consistent with the result that the effect of rest periods will be very small, if any.

7.5c. The environmental effect.

The chemical environment of a component in an aircraft structure differs from the conditions in a laboratory test. The time of exposure will be longer and the chemical activity of the environment may be different. Now in an aircraft structure special measures are taken to protect it against severe corrosive conditions. A general more or less systematic discussion is hardly possible and beyond the scope of this report. Reference may be made to a recent NACA investigation (ref. 32) on sheet specimen of 2024 and 7075 - material both in the bare and clad condition, notched by a central hole, loaded in plane bending, $k_t = 1.6$. For each material one batch of specimens was tested uninterruptedly indoors at a speed of 575 cycles per minute and another batch was tested outdoors at a speed of 430 cycles per minute and 10 minutes per day only. The latter specimens were freely exposed to rain and moist. The outdoor environment was situated at the Atlantic coast of the U. S. A. and was supposed to be quite severe compared with average conditions experienced by aircraft. The specimens were tested at $S_m = 8.4$ kg/mm². The following results were obtained.

Material	S_a (kg/mm ²)	N^1		Reduction factor
		Indoors	Outdoors	
2024-T3 clad	10.5	541000	546000	0.99
7075-T6 clad	10.2	389000	280000	1.39
2024-T3 bare	17.6	399000	145000	2.75
7075-T6 bare	17.6	211000	90400	2.33

1) Logarithmic mean of about 25 specimens. Ref. 32 cites arithmetic means.

It turns out that for the clad material the environmental effect is fairly small (7075) or negligible (2024). For the bare material there is a noticeable unfavourable effect.

A large part of the fatigue fractures occurring in a structure start at rivet-holes or bolt-holes and the accessibility of the critical location of these types of notches for any liquid substance will be poor. An important effect of the environment should not be expected, then.

A different situation exists if a fatigue crack is growing in a structure. In general the accessibility for the environmental medium will be fairly good. Little is known about the effect of the environment on crack propagation and it is felt that empirical research on this topic is highly advisable.

Another aspect is the possibility of stress-corrosion to which some light alloys seem to be liable. If this phenomenon occurs in combination with fatigue, a rather unfavourable situation exists. This, however, will not be discussed in this report.

7.5d. The load sequence in a program-fatigue test and in service.

The difference between program loading and service loading will be discussed on the assumption that there is no effect of the rate of loading, rest periods and environment. The problem is then whether the load variations as they occur in practice may be reduced to a load sequence as applied in program-fatigue tests. To deal with this question two well-known methods for the reduction of a load-time history will be briefly discussed. The methods will be called the peak-method and the range-method. They are illustrated in fig. 51. The peak-method is associated with the so-called "Fatigue-meter" (refs. 51 and 61). From the indications of this instrument the number of peaks in a certain load interval can be deduced. * A peak of the load-time history is an absolute maximum above the mean load or an absolute minimum below the mean load. The combination of 2 equal peak loads with opposite sign to one complete load cycle disregards that these peak loads do not always start from or return to the mean load, see for instance peak A and B in fig. 51. Further, the method disregards that in general 2 equal peak loads of opposite sign do not immediately succeed each other. The peak-method suggests load variations which actually do not occur. Especially, too many large load variations will be found. The method implies a rather severe violation of the actual load-time history.

The range-method is based on the assumption that in general load variations are more significant than peak loads or the mean load (ref. 52). This method analyses the load ranges, i. e. the differences between successive peaks

and troughs of the load-time history. As an approximation all load ranges are supposed to have the same mean load. Equal load ranges of opposite sign are combined to complete load cycles. The distribution of load cycles obtained by this method involves load variations which have actually occurred. Only the sequence is violated and the effect of variations of the mean load is disregarded. The range method may be associated with the Strain-Range-Counter (ref. 62). **

Two pertinent questions may be raised now:

- (1) Which of both methods is the most severe one?
- (2) Which of both methods will involve the best simulation of the load sequence occurring in service?

There is no obvious answer to the first question. The range method will give more cycles but the cycles with the higher amplitudes will be less numerous. Calculations of Payne (ref. 37) see fig. 37, for a random load test showed that the range-method (hypothesis H_2) gives a longer life than the peak-method (hypothesis H_1 and H_3). They suggest that in this case the peak-method was the most severe method, which probably is in accordance with intuitive expectations. With respect to the second question there are no physical arguments available to judge which method will give the best simulation of service history. However, one may consider the question which method will give conservative information. Conclusive answers have to come from tests. Until now only one study on this point has been published which is due to Kowalewski (ref. 30). A summary of his tests is presented in fig. 47. Kowalewski employed random noise which he analysed with the peak and the range-method both. He then compared the fatigue life in tests with random loading and program-loading with a program of peaks and a program of ranges. Moreover, Kowalewski has made this empirical comparison for random noise with different degrees of irregularity. A convenient indication of the irregularity is the ratio of the number of zero-crossings per unit time (N_0) and the number of absolute maxima in the same time (N_1). For $N_0/N_1 \sim 1$ the load-time history has the appearance of a sine wave with a modulated amplitude, see fig. 47. For such a load sequence the peak- and the range-method

* In principle the number of peak loads cannot be deduced from the numbers of level crossings, which are counted by the Fatigue-meter. However, the result obtained will not be much different from the results desired.

** Actually the Strain-Range-Counter does not exactly count ranges in the sense as discussed here.

will give the same reduced load distribution. The value of N_0/N_1 does not uniquely determine the irregularity. The complete statistical information is contained in the power spectrum, see for instance ref. 42.

Fig. 47 shows that the program of peaks gives a life which is about half the life under random loading, irrespective of the N_0/N_1 -value. The program of ranges also gives a higher fatigue life than the program of peaks, the difference being larger for higher degrees of irregularity. This may be explained by consulting fig. 52. A load cycle with some small intermediate interruptions (not seldom for a high degree of irregularity) will be reduced by the range method to 2 cycles with half the amplitude of the original one. Increasing the number of cycles twice accompanied by halving the amplitude will imply a less severe loading. Kowalewski's results learn that it may be an unsafe procedure to employ the range-method since it may give a longer life than for irregular types of load-time histories.

Two questions arise from the previous discussion.

- (a) How irregular are alternating loads in service?
- (b) Is it allowed to generalize Kowalewski's results?

With respect to the first question it has to be remembered that the irregularity of the loading of an aircraft structure depends not only on the randomness of the external loadings, but it is also largely governed by the response of the aircraft. A general answer is impossible. By strain gage measurements in flight one may gain a quantitative impression of the irregularity and for fatigue critical parts of an aircraft such measurements are highly advisable anyhow. The gust loads have been studied in most detail (refs. 40 and 41). It seems that under constant weather conditions these loads behave as a stationary Gaussian process, just as random noise. It implies that the irregularity of the gust load is fully described by its power spectrum. Additional problems are the variations in weather conditions (ref. 41) and the response of the aircraft (ref. 42). A detailed discussion is beyond the scope of this report.

Now let the optimistic assumption be made that the degree of irregularity is known, what can then be said about the second question (b). Obviously, no exact general validity can be attributed to Kowalewski's results, as a backing by a physical theory would be a prerequisite. At first sight Kowalewski's test conditions look fairly restrictive. He employed a mild notch ($k_t = 1.77$), plane bending with $S_m = 0$ and random noise. Moreover, previous sections have learned that program-fatigue testing, with which the comparison was made, does not

guarantee a unique fatigue life, depending on the distribution of the load amplitude only. There may be some influence of the way of programming.

The density distribution function of the peaks of random noise is approximately a Rayleigh distribution except for small values of the peaks. The Rayleigh distribution is a fairly poor approximation for an average distribution of gust-peak loads. For usual root-mean-square values it will contain relatively too many high loads and too few low loads. It must be strongly advised to extend the research initiated by Kowalewski to broader testing conditions. It is still believed that Kowalewski's conclusion that an irregularly varying load will produce a longer life than the corresponding sequence of peak loads is qualitatively correct. This belief is founded on the method of reducing the irregular load-time history to peak loads as discussed in this chapter and the view that a random sequence will probably allow a better periodic restoration of favourable residual stresses.

- 7.5e. Recapitulation of the value of program-fatigue testing as a means of simulating service loading.

The following relates to the fatigue life until technically visible cracks only. From the previous sections it will be clear that there is a large difference between program-fatigue testing and service loading. In general the loading rate will be higher in a fatigue test than in service. It is felt that this speed effect will be very small. Secondly rest periods are not introduced in laboratory tests to such an extent as in service. Also this may have a slight effect. The chemical effect of the environment in service, if noticeable, will be unfavourable in comparison with the laboratory test. Again it is thought that in many instances the effect will be small. Comparing the load sequence in service and the load sequence in a program-fatigue test the latter sequence is felt to be a conservative representation of the former if the reduction of the load history in service has been made with the peak-method. There is no obvious better or best method for such a reduction. The range-method may give unconservative results.

The total impression is that adding the favourable and unfavourable effects the program-fatigue test can be a conservative representation for service loading provided that the program is established with some care as outlined in section 7.2. This means that loads below the estimated fatigue limit are included, an increasing-decreasing or a randomized load sequence is used, at least 10 periods will be involved in the test, extremely high loads are avoided and ground-to-air cycles are included if they may be important.

The proof of the pudding would be a comparison of service experience with laboratory tests. Such comparisons of sufficient quality are hard to obtain because they require that the service loading of the aircraft and the loading of the particular component in its structure are well established. Gassner (ref. 20) reports the failure of a riveted joint in a wing spar of the Ju-52 aircraft, which afterwards was tested in a program-fatigue test. The result of the test differed only 10% from the service life and the same type of failure was obtained. Huston (ref. 9) could make a comparison for the C-46 aircraft. The results are summarized in fig. 41. Also here the agreement is reasonable, somewhat larger lives being obtained in service. For the Ju-52 as well as the C-46 the majority of damage will have been due to gusts. Under such circumstances ground-to-air cycles may be assumed to be more damaging than suggested by the Palmgren-Miner rule. These loads were not introduced into the program-fatigue tests and nevertheless a fair agreement was reached. This is consistent with the view that program-fatigue testing may give conservative life estimates. In ref. 60 the fatigue life obtained in service for a wing skin joint was compared with the life calculated from the relevant S-N-curve with the Palmgren-Miner rule. A summary is given in fig. 42. Also here no account was taken of the damage of the ground-to-air cycles; the service life was somewhat longer than the calculated life. Finally Gassner (ref. 20) claims that for motor-car components (steel) the fatigue lives in practice are twice as long as the fatigue life in the corresponding program-fatigue tests. The value of the above-mentioned evidence is not large since there are too many uncertain parameters involved. It is highly desirable therefore that more information becomes available for aircraft for which the service loading has been well established.

7.6. The value of the Palmgren-Miner rule for the designer.

The Palmgren-Miner rule is essentially a rule for making life estimates. It presupposes the availability of relevant S-N-curves. Questions covering different aspects are:

1. What is the purpose of the estimate,
 - (a) a life prediction until visible cracks or
 - (b) a life prediction of the crack stage?
2. What is the quality desired for the life estimate,
 - (a) a rough estimate or
 - (b) an estimate as precise as possible?
3. At which moment will the estimate be made,
 - (a) in the design stage or
 - (b) after the design has been frozen?

All these questions have a rather intimate relation with each other. A detailed discussion

would take too much room here. However, to indicate the implications of the present investigation and the literature review on similar investigations a brief discussion will be presented. This remark also applies to the following section 7.7.

Ad 1(a)

The evidence presented in this report suggests that a prediction of the life until visible cracks for a light alloy structural component for which the mean stress is positive, based on the Palmgren-Miner rule, will give a result which on the average will be on the safe side. Unconservative estimates will be promoted by many low loads, below the fatigue limit, in the absence of reasonably high positive loads. Unconservative estimates will also be promoted by periodic negative loads, such as ground-to-air cycles, which are not counterbalanced by equally numerous high positive loads. In any case the damage of periodic negative loads should not be calculated separately, but in connection with all the other loadings combined to one load spectrum. If load-time registrations are available they should be reduced to a load spectrum with the peak-method and not with the range-method. It is felt that the former will be a safe procedure whereas the latter might be unsafe. The countings of the Fatigue-meter may be considered as a fair approximation of a reduction according to the peak-method.

Ad 1(b)

There is not much empirical evidence on life predictions for the crack stage or the crack rate. The impression is that also here the Palmgren-Miner rule can safely be used but more research is certainly advisable.

Ad 2(a)

What is a rough estimate of an endurance? It is felt that any estimate from which it may be expected that it will yield the right order or magnitude will fall in this category. From this point of view the Palmgren-Miner rule can certainly be regarded as a useful rule.

Ad 2(b)

Difficulties arise when a precise life estimate is wanted. A precise estimate should not be confused with a safe life estimate. Factors of safety and related problems are not discussed in this report. It is felt that a life estimate which does not err more than a factor of 2, i. e. $\sum \frac{1}{N}$ between 0.5 and 2, may be regarded from a technical point of view as a good job. If this view is accepted inaccurate life estimates which are too high ($\sum \frac{1}{N} < 0.5$) will be very seldom. Inaccurate life estimates which are too low are possible. No definite rules can be given here and some guidance may be found in the surveyed empirical data. Now it should

not be overlooked that a precise life estimate depends not only on the usefulness of the Palmgren-Miner rule, but perhaps still more on the accuracy of the employed S-N-curve, the knowledge of the load spectrum and the loading of the particular component in a structure. This virtually excludes the possibility of very precise life estimates.

Ad 3(a)

The Palmgren-Miner rule is felt to be useful as long as the local dimensions of joints, cut-outs and other stress raisers still allow some modifications. If a reasonable estimate of an S-N-curve and a load spectrum can be obtained, no cumbersome problems will arise if a life estimate with the Palmgren-Miner rule will give a very short or a very long life. In the first case local dimensions or geometry will be changed and in the latter case no design action need be taken. For the design action to be taken in intermediate situations, i. e. when the part under consideration may be suspected of being fatigue critical, a number of possibilities exist. For instance, (1) one may still change the design locally if this is easily possible, (2) if the structure has obvious fail-safe characteristics one may accept the risk of an insufficient life, (3) one may try to improve the quality of the life estimate, for instance by ad hoc fatigue tests. This problem will not be discussed further here since there are also economical aspects involved.

Ad 3(b)

Once the design has been frozen, i. e. when modifications have undesirable production implications, a life estimate which does not yield an acceptable long life or a life estimate of unacceptable quality, should be a strong argument for fatigue testing the part under consideration.

A thorough discussion of the fatigue problem of the designer is beyond the scope of this report. The conclusion to be drawn here is that the Palmgren-Miner rule is a useful tool for the designer. The use of the rule, however, is quite limited, due to uncertainties which have nothing to do with the rule itself. One important factor may be recalled here. In general no directly applicable S-N-curve will be available. In many cases the Palmgren-Miner rule will therefore not exclude the need of ad hoc fatigue tests.

Finally reference may be made to alternative theories which have been suggested in the literature (refs. 8, 14, 15, 27, 31 and 48). Similar to the Palmgren-Miner rule these theories are missing a clear and decisive physical background. They are almost invariably checked with tests on unnotched specimens for which they predict the occurrence of $\sum \frac{1}{N} < 1$. The theories involve at least one empirical parameter which obviously increases their flexibility. However,

this parameter probably will depend on some of the fatigue parameters (S_m, k_t , material, etc.). Their usefulness for practical problems has still to be proven and this will involve a large amount of testing. For the time being it is advised to give preference to the simpler Palmgren-Miner rule. The suggestion of Smith (ref. 49) to use the Palmgren-Miner rule in combination with the S-N-curve of specimens preloaded until the highest load of the spectrum, see Fig. 31, will in general yield higher life estimates, due to the favourable effect of the preload on the S-N-curve. It is not certain that it will always give sufficiently conservative results. The favourable residual stresses formed by the preload will be relaxed under a fatigue loading but it is difficult to see that this will always occur in the same way irrespective of the type of fatigue loading. In fact the results of the present investigation suggest that such is not the case. Further, most available S-N-curves have been determined for not preloaded specimens. In view of these arguments the usefulness of Smith's proposal is doubted.

7.7. The value of program-fatigue testing for the designer.

Performing program-fatigue tests on a component or a structure is essentially an empirical approach to obtaining information on its fatigue characteristics. Different aspects are:

1. The purpose of the test may be
 - (a) a life estimation or a crack-rate estimation
 - (b) revealing the fatigue critical parts.
2. What is the quality of the information to be obtained?
3. How does the method compare with the following alternatives:
 - (a) Fatigue tests to determine the relevant S-N-curve(s) and employing the Palmgren-Miner rule.
 - (b) The R. A. E. -test.
 - (c) Random-load test.

Ad 1(a)

A life estimation may be the basis for retirement times for those components which are not "fail-safe" or which allow an easy replacement. The crack-rate information may serve to establish inspection periods.

Safety factors to be applied will not be discussed. Obviously such factors will have to depend on the quality of the information.

Ad 1(b)

Revealing the fatigue critical components of a structure is one of the most important potentialities of a full-scale test on a structure. This type of information can hardly be obtained by theoretical reasoning.

Ad 2

The quality of the information depends on a number of questions.

- (a) How representative is the component or structure to be tested?
- (b) How representative is the assumed load spectrum?
- (c) How is the load spectrum reduced to a programmed load sequence?

- (a) It is clear that the structure should be as representative as possible with respect to the production type. So it should include normal production methods and it should be full scale to avoid size effects which are not yet fully understood.
- (b) It is highly important that all types of alternating loads which may occur in service are well represented. This is not only important with respect to the fatigue life estimation but perhaps even more with respect to indicating possible fatigue critical locations. Small but numerous alternating loads may introduce fatigue failures due to fretting corrosion in areas where the nominal stress is still fairly low.
- (c) Some remarks on arriving at a programmed load sequence have been made in section 7.5e and will not be repeated here. In addition shorter periods have to be advised for the crack-propagation stage if it must be expected that this stage will cover a small part of the fatigue life.

Ad 3

Program-fatigue testing has advantages and disadvantages. They will be discussed by comparing this testing method with alternative methods.

Ad 3(a)

One alternative is the determination of the relevant S-N-curve(s) and a subsequent life calculation with the Palmgren-Miner rule. For this alternative it is assumed that no S-N-curves of sufficient quality can be derived from the literature and that such curves have to be established by tests. There are two obvious aspects then. The alternative implies (1) much simpler testing techniques and (2) much more specimens. The latter argument will be a prohibitive reason to apply this alternative to structures. There are other important aspects, however. (3) The mode of failure in constant amplitude tests may depend on the magnitude of the load amplitude and this complicates the interpretation with respect to indication fatigue critical components. Valuable experience in this respect was obtained with the Commando wings (refs. 29 and 59) see figs. 40 and 41. Similar evidence was reported by Haas (ref. 24). The evidence is certainly in favour of the pro-

gram-fatigue test. A fourth aspect is the flexibility of the alternative method which in principle is greater than for program-fatigue testing. The result of the latter method applies specifically to the load spectrum applied in the test and the spectrum in service may turn out to be somewhat different. However, corrections can easily be made with the Palmgren-Miner rule if S-N-curves are available. The quality of these corrections is nothing better than the quality of this rule, see section 7.6.

It is believed that corrections to the result of a program-fatigue test with the aid of the empirical data as summarized in this report, will be acceptable. So the alternative method 3(a) cannot be considered to be a serious competitor of program-fatigue testing. The drawback of the latter method, viz. the more laborious testing technique, is fully balanced by the much smaller number of specimens and the more exact information.

Ad 3 (b)

The RAE-test was developed at the R. A. E. (ref. 1) and until now it has been applied for a number of aircraft. Originally the loads applied were (1) gust loads, (2) ground-to-air cycles and (3) pressurisation cycles on the fuselage. Almost the complete aircraft structure is involved in the test. A special characteristic is that all gust loads are reduced to one level, the estimated level of maximum damage, see fig. 10. This occurs with the aid of a more or less relevant S-N-curve and the Palmgren-Miner rule. The reduction is performed in such a way that the one-level gust loading involves the same damage as the complete anticipated gust spectrum. Another peculiarity is that for the application of the pressurisation cycle the fuselage is immersed in a water tank. The load sequence is illustrated in fig. 53. The result is very often expressed as the number of simulated flights. The discussion in previous sections allows the following comments.

- (1) It is unlikely that the fatigue life and the crack period of the wing will be overestimated if the correct S-N-curve can be used for reducing the gust spectrum. This conclusion seems to be allowed since the unfavourable ground-to-air cycles are incorporated. The idea of Barrois (ref. 2) to include also the positive gust load which should be anticipated once per flight, see fig. 28, is probably a further improvement. The RAE-test has one inherent weak point. The reduction of a load spectrum to one level requires an S-N-curve and it can not be expected that this curve will be valid for all the components of a structure. It can easily be shown that a horizontal shift of an S-log N-curve will not affect the reduction to one level. However, the reduction is affected by the slope of the curve and

even more by the fatigue limit. This makes a life estimate obtained by this type of test a fairly debatable result.

- (2) The quality of the information to be obtained with the RAE-method depends on the question whether all important fatigue loadings have been introduced in the test or have been accounted for in some other way. Obviously this remark applies to program-fatigue testing as well.
- (3) By employing a progressive repair-technique, i. e. by repairing all failures encountered in the test and continuing the test after repairs, it will be possible to reveal many of the fatigue critical parts with the RAE-test. In this respect the RAE-test has shown its usefulness for a number of aircraft, also allowing experience to be gained concerning repair techniques and inspection methods. It has been said previously that the locations of failures in a structure will depend on the load amplitude. Therefore it has to be doubted whether the RAE-test will reveal the critical locations in the correct order and whether it will reveal all critical parts. In this respect the selection of the gust loading or the level of maximum damage cannot be considered to be a rational excuse since the adjective "maximum" is more or less fictitious in view of the non-validity of the Palmgren-Miner rule. In the RAE-tests those parts where many low-amplitude cycles may induce fretting corrosion followed by fatigue crack initiation, may be overlooked. Also from this point of view of indicating critical components program-fatigue testing should certainly be preferred. The view is sustained by the experience with the Commando wings recently reported by Huston (ref. 29) Haas (ref. 24) made similar observations.
- (4) The RAE-test has one obvious advantage over program-fatigue testing which is its simpler testing technique. Whether this argument will fully outweigh the draw-backs in the future is subject to severe doubt now that the possibilities of testing apparatus are steadily improving.

Ad 3(c)

A random-load test has to be considered as a further refinement of a program test. It will involve a certain type of randomness, which need not be the randomness occurring in service. For instance, for the tests on the Mustang wings, the only random-load test performed on a structure until now, the gust loads all return to the 1-g-load, see fig. 37. Such a sequence will certainly not continuously occur in service. Still it is felt that the sequence showed more resemblance to the load-time history in service than a programmed sequence would have done. With the evidence now

available it is difficult to appreciate the improvement of the information to be gained with a random test.

More research in this field may be advised. Obviously the random test implies a more complicated test set-up than a program-fatigue test. Probably this need not be an insuperable objection.

Attention may be drawn here to the randomized-step tests as applied to the Commando-wings, see fig. 40. Actually such a test has still to be considered as a program-fatigue test; however, the load steps of a period are mixed at random. It is believed that this is an improvement, again at the cost of a more complicated load-monitoring system. Also here the real improvement is difficult to judge.

Summarizing this section it may be concluded that program-fatigue testing a structure or a component will provide the designer with useful information concerning life estimates, crack rate estimates and the indication of fatigue-critical locations. The quality of the information depends on the representation of the important fatigue loadings in the test. From the alternative testing methods it seems (1) that a determination of S-N-curves combined with the use of the Palmgren-Miner rule is a far inferior method (2), that the RAE-test will provide useful information, but is still inferior to program-fatigue testing and (3) that random-load tests and also randomized-step tests may be considered to be further improvements of program-fatigue tests for which the improvement is still difficult to judge.

7.8. Scatter

A few remarks will be made on the scatter encountered in the present investigation and its implications.

For the fatigue life (N) of the constant-amplitude tests and the total number of cycles (N') in the program-fatigue tests the standard deviation and the coefficient of variation have been calculated based on the logarithms of the test results, see tables 1, 2, 5 and 6 and fig. 18. Fig. 18 shows a remarkable result. For the constant-amplitude tests scatter increases with increasing endurance, which was already known, see ref. 46. For the program-fatigue tests, however, the values of the coefficient of variation are scattered around a mean value of about 1.5% without revealing a systematic influence of the endurance N' .

Potential sources of scatter were felt to be

- (a) The fatigue machine.
- (b) The execution of the test.
- (c) The material of the specimen.
- (d) The geometry of the specimen.
- (e) The type of loading.

It will be difficult to discriminate fully between these sources. However, the systematic trend of fig. 18, i. e. increasing scatter for constant-amplitude tests and constant scatter for program-fatigue tests obviously indicates source (e) as a dominating source. It also shows that source (e) has not been overshadowed by the other sources (a) to (d), which may be interpreted as an indication that the effect of these sources has not been excessive.

With respect to the effect of the material on scatter Gassner (ref. 17) found that test results at the upper side of the scatter band correspond to specimens with a low yield point. Systematical evidence was offered by Fisher (ref. 9). He tested joints of nominally identical material, viz. an extruded light alloy. For the same fatigue loading the endurance of the joint increased with decreasing values of the ratio $S_{0.1}/S_u$. From the present test program 9 test series were selected with a relatively large scatter. From the specimens with the longest and the shortest life of these groups a tensile specimen was cut from the sheet in which the fatigue failure occurred.

For the specimens $S_{0.2}$, S_u , $S_{0.2}/S_u$ and the elongation were determined. None of these quantities showed any correlation with the fatigue-life.

Now drawing once again attention to fig. 18, the figure shows that the scatter in the program-fatigue tests is of the same magnitude as for constant-amplitude tests with the highest stress amplitudes applied in the program-fatigue tests. This suggests, without proving it that the highest stresses may have a decisive effect on the scatter. Similar results were obtained by Wallgren (ref. 55), see fig. 26, who showed that the scatter in program-fatigue tests is much lower than predicted with the Palmgren-Miner rule on the basis of the scatter in the S-N-curve. This is consistent with the trend of fig. 18. Also Gassner (ref. 22) showed that the scatter in program-fatigue tests on notched rotating-beam specimens of 2024 material was of the same order as for constant-amplitude tests at the highest stress level of the program. Testing a joint Haas (ref. 24) found a lower scatter for program-fatigue tests than for constant-amplitude tests. For unnotched rotating-beam specimens Freudenthal and Heller (ref. 14) found similar results, i. e. higher scatter for constant - amplitude tests.

As a possible explanation the following suggestion may be given. It is not unlikely that the effect of inhomogeneities, whether they occur in the material, the load distribution over the rivets or the uneven clamping of the rivets will be smoothed by the highest loads of the program to a level corresponding to those loads in constant-amplitude tests.

With respect to scatter in fatigue trouble to be anticipated in service there seems to be one implication in the present investigation. Since program-fatigue tests show less scatter than constant-amplitude tests with a comparable endurance, it is felt that scatter in service cannot be predicted directly on the basis of constant-amplitude tests on components in the laboratory. No more comments will be given here on this rather delicate question. The most valuable information to be obtained in this respect has to come from service experience. It is hoped that such information will be published as much as possible. Until now few data were presented in the literature. One of the reasons will be that as soon as a fatigue crack has been found in some aircraft, action will be taken to eliminate its occurrence in other aircraft.

8. Summary and conclusions

In this report the results of program-fatigue tests on riveted joints of two light alloys are presented. The results of similar investigations in program-fatigue testing of notched light alloy specimens and structures are summarized. A discussion is then presented on the following questions:

- (a) How is the endurance in a program-fatigue test affected by the different parameters involved in establishing the load sequence for such a test? Some parameters are: Lowest and/highest stress amplitude to be included, number of cycles and load sequence in each period.
- (b) How is the fatigue life in a program-fatigue test affected by very high positive and/or negative loads which are applied at periodic time intervals?
- (c) What can be said about the validity of the Palmgren-Miner rule, $\sum \frac{D}{N} = 1$ or systematical deviations from this rule with reference to program-fatigue loading?
- (d) How does scatter in program-fatigue tests compare with scatter in constant amplitude tests?

In view of arriving at the technical implications of the foregoing questions for the designer some attention has been paid to:

- (e) What is the usefulness of the Palmgren-Miner rule for the designer?
- (f) How good is program-fatigue testing as a representation of the load-time history in service?
- (g) What information may be expected from program-fatigue testing and how does it compare with the information of alternative methods?

The field of interest is rather broad, although it has been restricted to the Palmgren-Miner rule and program-fatigue testing. It has been

tried to summarize the results of the discussion in a number of conclusions. All conclusions are restricted to notched light alloy specimens loaded at a positive mean stress. In a previous study a significantly different behaviour for unnotched and notched specimens has been revealed. The majority of the reviewed test data was concerned with testing according to gust spectra and maneuver spectra. Sonic fatigue is not considered in this report. In making use of the conclusions for a particular problem due consideration has to be given to the existence of special circumstances, not discussed in this report, which might invalidate the applicability of the conclusions. It will always be necessary to consult the evidence on which the conclusions were based. For this purpose the results of all reviewed investigations have been summarized in a number of figures. The conclusions do not intend to raise the impression that a reasonable level of knowledge has been gained from the point of view of the designer. On the contrary, this study has once more revealed many of the weak points in the present state of the art. Rather the conclusions should be looked upon as trends borne out by the fairly large amount of test results. At the end of the conclusions there is a list of problems which are considered to be of primary importance with respect to predicting fatigue properties in service.

Some data on the present investigations are The specimens were riveted lap joints with 2 rows of 8 rivets each. Sheet materials were 2024 Alclad and 7075 Clad. All tests were performed at a constant positive mean stress, which was 9.0 kg/mm² (12.8 k. s. i.) and 6.3 kg/mm² (9.0 k. s. i.) for the 2024 and 7075 specimens respectively. Relevant S-N-curves were determined with 40 and 50 specimens respectively. 24 test series of 7 program-fatigue tests each were performed. The program for these tests was associated with a gust spectrum.

On the parameters of the program.

1. The sequence of the load levels in a period may have a significant effect on the endurance. Extreme results have been obtained for increasing load levels only or decreasing load levels only. Increasing levels followed by decreasing levels in each period and randomized sequences have given intermediate results.
2. For the number of load cycles in a period only a small effect on the endurance has been found. A very large number of cycles per period, equivalent to a small number of periods until failure, may give low endurances.
3. Including many cycles with a low amplitude

below the fatigue limit may lower the fatigue life expressed as $\sum \frac{n}{N}$.

4. The cycles with the highest stress amplitude of the program have a favourable effect on the life expressed as $\sum \frac{n}{N}$.
5. The effect of the number of load levels in each period is probably small if this number is not too small.

On periodic high loads.

6. Very high loads inserted once in a period, exert a considerable influence upon the endurance, which is favourable for positive loads and unfavourable for negative loads. If such periodically applied high loads are immediately followed by an equally high opposite load this latter one has a dominating effect on the endurance.

On the validity of the Palmgren-Miner rule $\sum \frac{n}{N} = 1.$

7. A general validity of the Palmgren-Miner rule does not exist. Some systematic deviations of the rule can be indicated.
8. Program-fatigue tests on small notched specimens loaded at positive mean stresses have, in general, given $\sum \frac{n}{N} > 1$, with an average of 1.5-2.0. This seems to hold for 2024 and 7075 specimens both and also for gust-spectrum loading and maneuver-spectrum loading. Low $\sum \frac{n}{N}$ -values are promoted by zero mean stress, low stress-concentration factors, periodic negative loads (ground-to-air cycles) and many cycles with a low amplitude. This does not imply that such testing conditions will necessarily lead to $\sum \frac{n}{N} < 1$. High $\sum \frac{n}{N}$ -values are promoted by positive mean stress, cycles with high stress amplitudes and periodic very high positive loads. More quantitative guidance with respect to deviations of the Palmgren-Miner rule may be found in the surveyed data.
9. A qualitative explanation of the previous conclusions can be given on the basis of the formation, the relaxation and the restoration of residual stresses; with respect to the damage contribution of load cycles below the fatigue limit it is assumed that such load cycles which are unable to produce cracks themselves will assist in crack growth.
10. It is felt that the experience with small specimens will be applicable to structures for the life until visible cracks appear.
11. For structures a limited amount of experimental evidence is available. The information is obscured by the simultaneous occurrence of different interfering modes of failure. The general impression is that the Palmgren-Miner rule will be conservative

for structures including the crack-propagation stage.

12. For life calculations for which ground-to-air cycles are important in addition to a symmetrical gust-spectrum the damage of these cycles should not be calculated separately but they should be treated as being negative gust loadings, thus implying an asymmetric load spectrum.

On scatter.

13. For the same number of cycles until failure the scatter in program-fatigue tests is considerably lower than for constant-amplitude tests. The scatter for program-fatigue tests was of the order of the scatter for constant-amplitude tests at the maximum load level of the program, suggesting a dominating effect of that level with respect to scatter.
14. For the present investigation the coefficient of variation of the logarithm of the endurance in cycles for the program - fatigue tests showed a tendency towards an average value of 1.5% independent of the endurance. The standard deviation of the same property, excluding the test series with periodic high loads, had a mean value of 0.105.
15. In general constant-amplitude tests at the so-called "most damaging level" are not suitable to obtain indications about scatter to be expected under a more or less random loading.

On program-fatigue tests as a representation of service loading.

16. In comparing program-fatigue tests with service loading obvious differences are related to the speed of loading, to rest periods, to the environment and to the load sequence. It is believed that for the time until crack growth no large effect of speed, rest periods and environment need to be feared, provided that sufficient measures against corrosion have been taken and that no danger of stress corrosion exists. The available evidence then suggests that a carefully planned program-fatigue test will be a conservative replacement for service loading with respect to the life until crack growth; however, the evidence is still rather scanty.
17. It may be expected that in program-fatigue tests with an increasing-decreasing load sequence more damage will be done by low-amplitude cycles than in the corresponding random-load test.

On the usefulness of the Palmgren-Miner rule

18. In the design stage of an aircraft the Palmgren-Miner rule may be considered to be an acceptable means to obtain a first esti-

mate of the fatigue life. If all the data on which the estimate is based are correct unconservative estimates will be very seldom for notched parts loaded with a positive mean stress. Uncertainties may be introduced, however, due to insufficient knowledge on the anticipated load spectrum, the loading of the particular component in the structure and the S-N-curve to be used. The life estimate may be helpful in deciding whether any design action from the fatigue point of view or ad hoc fatigue tests are necessary.

On the information to be obtained from program-fatigue tests.

19. Program-fatigue tests will provide the designer with useful information concerning life estimates, crack rate estimates, the indication of fatigue-critical locations and inspection methods. The quality of the information depends on the representation of the important fatigue loadings in the test. Especially it should not be overlooked that "ground-to-air" cycles may be more damaging than suggested by the Palmgren-Miner rule.
20. With respect to the quality of information to be obtained from alternative testing methods it may be said (1) that an experimental determination of the relevant S-N-curves combined with the use of the Palmgren-Miner rule is an inferior method, (2) that the RAE-test will provide useful information, but is still inferior to program-fatigue testing and (3) that random-load tests in which a representative load-time history is applied will be a superior method. The degree of superiority of the latter method, however, is still difficult to judge and the method involves serious experimental complications.

Problems on which more research is desired.

21. A number of problems on which more research is highly desirable are listed below. The list is not meant to be complete, nor is it claimed that a sufficient knowledge exists of related problems which are not mentioned. It only attempts to indicate the most urgent problems associated with the subject discussed in this report.
- (a) Comparison of service loading with program-fatigue loading for notched elements and, if possible, structures.
 - (b) Study of fatigue-crack propagation in structures or representative components under program-fatigue loading and/or random loading in comparison with constant-amplitude loading.

- (c) The effect of frequency on crack propagation in large parts.
- (d) The effect of corrosive environments on crack propagation.
- (e) Scatter in fatigue trouble occurring in service.

- (f) The comparison of full-scale laboratory fatigue-tests with service experience for aircraft for which the service-load statistics are reasonably well known.

The National Aeronautical Research Institute will be grateful to receive any information on these topics.

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Table 1. Results of constant-amplitude fatigue tests for 7075-specimens at

$$S_m = 6.30 \text{ kg/mm}^2$$

Specimen ¹⁾	Sheet thickness (mm)	S_a ²⁾ (kg/mm ²)	\bar{S}_a (kg/mm ²)	N (kc)	$\overline{\log N}$	Mean life ³⁾ (kc)	$\sigma_{\log N}$	Variation coefficient
62	0.79	4.81		101				
133	0.77	4.93		104				
102	0.80	4.75		110				
33	0.82	4.63		115				
40	0.84	4.52	4.66	134	5.1168	130.8	0.0806	1.58 %
87	0.85	4.47		139				
154	0.81	4.69		142				
152	0.82	4.63		145				
180	0.80	4.75		165				
65	0.83	4.57		175				
24	0.81	3.78		145				
81	0.76	4.03		187				
4	0.85	3.61		225				
105	0.79	3.88		248				
140	0.87	3.51	3.76	260	5.3987	250.5	0.1155	2.14 %
130	0.83	3.70		281				
76	0.81	3.78		283				
6	0.82	3.74		284				
11	0.83	3.70		286				
5	0.80	3.83		382				
136	0.78	2.89		400				
83	0.80	2.81		400				
26	0.85	2.65		498				
161	0.81	2.78		566				
101	0.83	2.71	2.76	640	5.7932	621.2	0.1437	2.48 %
115	0.79	2.85		663				
52	0.82	2.75		675				
99	0.84	2.68		706				
125	0.82	2.75		759				
36	0.81	2.78		1236				
176	0.82	2.09		1197				
160	0.78	2.20		1218				
79	0.81	2.11		1658				
88	0.82	2.09		1868				
55	0.85	2.02	2.10	1889	6.3718	2354	0.3646	5.64 %
22	0.81	2.11		2038				
2	0.84	2.04		2455				
66	0.84	2.04		2599				
73	0.80	2.14		2847				
41	0.79	2.08		16533				

1) The specimens have been arranged in order of increasing life.

2) The load amplitude was constant for each group of specimens.

3) The mean life is the anti-logarithm of $\overline{\log N}$.

Table 2. Results of constant-amplitude fatigue tests for 2024-specimens at

$$S_m = 9.00 \text{ kg/mm}^2$$

Specimen ¹⁾	Sheet thickness (mm)	S_a ²⁾ (kg/mm ²)	\bar{S}_a (kg/mm ²)	N (kc)	$\overline{\log N}$	Mean life ³⁾ (kc)	$\sigma_{\log N}$	Variation coefficient
19	0.81	8.44		53				
112	0.78	8.77		61				
5	0.77	8.88		67				
43	0.80	8.55		67				
79	0.78	8.77	8.71	70	4.8454	70.05	0.0625	1.29 %
89	0.79	8.65		71				
68	0.81	8.44		73				
17	0.76	9.00		76				
52	0.79	8.65		79				
102	0.77	8.88		90				
139	0.81	5.33		184				
137	0.80	5.40		199				
61	0.80	5.40		202				
24	0.76	5.68		207				
13	0.77	5.61	5.50	207	5.3554	226.7	0.0638	1.19 %
142	0.79	5.47		234				
155	0.78	5.54		246				
44	0.78	5.54		255				
94	0.77	5.61		273				
22	0.79	5.47		282				
153	0.76	3.62		457				
157	0.78	3.52		498				
108	0.73	3.76		585				
136	0.79	3.48		592				
56	0.80	3.43	3.50	608	5.8107	646.7	0.1134	1.95 %
30	0.77	3.57		641				
26	0.78	3.52		663				
42	0.80	3.43		667				
23	0.79	3.48		802				
81	0.82	3.35		1173				
38	0.73	2.70		1239				
74	0.79	2.49		1345				
41	0.78	2.52		1519				
57	0.77	2.55		1574				
130	0.80	2.46	2.51	1628	6.3158	2069	0.2168	3.42 %
91	0.82	2.40		1883				
98	0.78	2.52		1954				
75	0.76	2.59		2602				
84	0.80	2.46		3962				
156	0.79	2.49		5847				
149	0.77	1.94		>3497				
28	0.79	1.89		3629				
135	0.77	1.94		3695				
103	0.76	1.96		3891				
125	0.79	1.89	1.90	4282	6.9076	8083	0.4787	6.93 %
18	0.78	1.91		4440				
46	0.81	1.84		7045				
6	0.80	1.86		17413				
152	0.78	1.91		>48779				
110	0.80	1.86		>57380				

- 1) The specimens have been arranged in order of increasing life.
- 2) The load amplitude was constant for each group of specimens.
- 3) The mean life is the anti-logarithm of $\log N$.

Table 3. Numbers of load cycles in one period for the program-tests on 7075-specimens.

Number of test series	Number of cycles (kc) in one period at S_{ai} ¹⁾ (kg/mm ²)							High loads in one period (kg/mm ²)		Load sequence shown in figure:
	S_{a1} =1.2	S_{a2} =1.60	S_{a3} =2.00	S_{a4} =2.75 ²⁾	S_{a5} =3.76	S_{a6} =4.67	S_{a7} =6.30	S_{a8} =11.80 ³⁾	S_{a9} =18.15 ⁴⁾	
1	-	-	96	48	13	5	-	-	-	14
2	-	-	49	26	6.5	-	-	-	-	14
3	-	-	45.6	22.8	6.5	6.5	-	-	-	14
4	-	-	134.5	21.5	6	2	-	-	-	14
5	-	-	48.5	24	6.5	2.5	-	1 ⁵⁾	-	17
6	-	-	48.5	24	6.5	2.5	-	1 ⁶⁾	-	17
6a	-	-	48.5	24	6.5	2.5	-	1 ⁷⁾	-	17
6b	-	-	48.5	24	6.5	2.5	-	-	-	17
7	-	-	48.5	24	6.5	2.5	-	-	1	17
8	-	-	48.5	24	6.5	2.5	-	1	1	17
9	-	-	48.5	24	6.5	2.5	-	1	1	17
10	-	-	47.5	24.3	6.5	2.7	0.5	-	-	14 to 17
11	-	81	47.5	24.3	6.5	2.7	0.5	-	-	14
12	-	-	24	12	3.25	1.35	0.3	-	-	15, 16
13	-	-	24	12	3.25	1.35	0.3	-	-	15
14	-	-	47	24	6.5	2.7	0.7	-	-	15
15	163	81.5	47.5	24.3	6.5	2.7	0.5	-	-	14
17	-	-	48.5	24	6.5	2.5	-	1	-	17
18	-	-	94.5	48.4	13	5.3	1.0	-	-	14, 16

1) In the program-tests the load amplitudes have been obtained by multiplying S_{ai} with nominal cross-section based on the mean thickness (0.815 mm).

2) $S_{a4} = 2.75$, except that in series 1 and 2 a slightly lower value has been used, i. e. 2.66 instead of 2.75. S_{a4} is the level of maximum damage.

3) S_{a8} was applied as one positive half load cycle per period.

4) S_{a9} was applied as one negative half load cycle per period.

5) In this test series only one high pre-load to $S_m + S_a = 23.1$ kg/mm² has been applied.

6) Positive high loads omitted after the 50th period ($\sum \frac{n}{N} \sim 4.65$).

7) S_{a8} was applied in odd numbered periods only.

Table 4. Numbers of load cycles in one period for the program-tests on 2024-specimens.

Number of test series	Number of cycles (kc) in one period at $S_{ai}^{1)}$ (kg/mm ²)						High loads in one period	Load sequence shown in figure:
	$S_{a1}=0.81$	$S_{a2}=1.51$	$S_{a3}=2.42^{3)}$	$S_{a4}=3.77$	$S_{a5}=5.12$	$S_{a6}=8.08$	$S_{a7}=13.52^{2)}$	
21	-	320	95	15	2.26	0.175	-	14, 15, 17
22	-	320	95	15	2.26	0.175	-	15
23	-	320	95	15	2.26	0.175	-	15
24	-	320	95	15	2.26	-	-	14, 17
25	402	320	95	15	2.26	0.175	-	14
27	-	320	95	15	2.26	0.175	1	17
28	-	320	95	15	2.26	-	1	17

1) In the program-tests the load amplitudes have been obtained by multiplying S_{ai} with the nominal cross-section based on the mean thickness (0.785 mm).

2) S_{a7} was applied as a complete load cycle starting with the positive half cycle in series 27 and starting with the negative half cycle in series 28.

3) S_{a3} was the level of maximum damage.

Table 5. Test results of program-tests on 7075-specimens.

Test-series	Specimen ¹⁾ number	Sheet thickness (mm)	$\sum \frac{n}{N}$	$\overline{\sum \frac{n}{N}}$ ²⁾	Total life ³⁾ in periods	N' = total life in cycles (kc)	$\overline{\log N'}$	$\overline{\sigma} \log N'$	Coefficient of variation of $\log N'$
1	15	0.81	0.639	0.762	4	643	5.9024	0.0606	1.03 %
	151	0.83	0.656		5	768			
	77	0.82	0.666		5	744			
	34	0.83	0.809		6	930			
	95	0.78	0.855		5	773			
	109	0.84	0.875		6	972			
	90	0.80	0.883		5	810			
2	58	0.85	0.463	0.713	9	706	5.9430	0.1704	2.87 %
	1	0.79	0.464		3	488			
	59	0.82	0.623		10	796			
	78	0.82	0.662		10	815			
	123	0.79	0.779		11	840			
	69	0.83	1.084		18	1448			
	25	0.81	1.249		18	1467			
3	108	0.82	0.655	0.866	6	489	5.8122	0.1015	1.75 %
	113	0.76	0.728		6	453			
	68	0.86	0.782		9	722			
	107	0.83	0.857		9	708			
	19	0.80	0.953		8	651			
	39	0.82	1.047		10	808			
	103	0.81	1.142		10	814			
4	63	0.81	0.406	0.587	4	656	5.9972	0.0915	1.53 %
	49	0.84	0.495		6	982			
	127	0.84	0.545		7	1073			
	168	0.82	0.574		6	984			
	131	0.78	0.645		6	955			
	164	0.81	0.710		7	1148			
97	0.79	0.831	8	1283					
5	18	0.85	1.162	1.417	16	1302	6.1351	0.0755	1.23 %
	94	0.80	1.213		14	1084			
	80	0.83	1.240		16	1295			
	128	0.82	1.309		16	1295			
	54	0.81	1.341		16	1261			
	31	0.82	1.646		20	1621			
	70	0.79	2.154		23	1825			

Table 5, Cont. 1.

Test series	Specimen ¹⁾ number	Sheet thickness (mm)	$\sum \frac{n}{N}$	$\overline{\sum \frac{n}{N}}$ ²⁾	Total life in periods ³⁾	N' = total life in cycles (kc)	$\overline{\log N'}$	$\sigma_{\log N'}$	Coefficient of variation of log N'
6	116	0.81	8.377	9.895	95	7734	6.9958	-	-
	53	0.83	10.085		126	10269			
	129	0.84	11.468		150	12225			
6a	156	0.80	5.156	5.511	56	4556	6.6866	-	-
	13	0.79	5.414		56	4556			
	114	0.79	5.667		59	4776			
	163	0.82	5.826		69	5625			
6b	23	0.80	1.348	3.677	15	1214	6.5451	-	-
	124	0.81	1.773		20	1630			
	157	0.85	2.879		40	3228			
	30	0.84	8.189		107	8722			
	118	0.78	11.926		117	9537			
7	85	0.85	0.415	0.512	6	483	5.7055	0.0509	0.89 %
	120	0.82	0.430		6	444			
	46	0.81	0.452		6	444			
	71	0.84	0.519		7	569			
	50	0.82	0.549		7	562			
	96	0.79	0.585		6	489			
	135	0.79	0.686		8	583			
8	51	0.85	0.352	0.695	5	406	5.8357	0.1025	1.76 %
	117	0.82	0.675		8	652			
	166	0.82	0.680		9	747			
	84	0.80	0.698		8	643			
	45	0.77	0.852		9	699			
	82	0.81	0.888		10	815			
	7	0.84	0.918		12	978			
9	138	0.83	1.852	3.114	24	1923	6.4633	0.1350	2.09 %
	158	0.82	2.448		29	2364			
	21	0.81	2.530		29	2355			
	121	0.76	3.601		32	2608			
	20	0.85	3.737		51	4155			
	104	0.81	3.843		44	3565			
	112	0.80	4.784		52	4229			

Tabel 5, Cont. 2.

Test series	Specimen ¹⁾ number	Sheet thickness	$\sum \frac{n}{N}$	$\overline{\sum \frac{n}{N}}$ ²⁾	Total life ³⁾ in periods	N' = total life in cycles (kc)	$\overline{\log N'}$	$\overline{\sigma \log N'}$	Coefficient of variation of $\log N'$
10	47	0.82	0.827	1.078	9	734	5.9549	0.0974	1.64 %
	100	0.82	0.877		10	806			
	17	0.84	0.882		11	887			
	153	0.80	0.969		10	744			
	37	0.87	0.987		13	1107			
	155	0.79	1.386		13	1083			
	119	0.75	1.994		16	1300			
11	14	0.85	0.568	0.910	7	1142	6.2012	0.1239	2.00 %
	159	0.78	0.731		7	1132			
	12	0.80	0.761		8	1294			
	72	0.83	0.922		11	1784			
	56	0.81	1.178		12	1956			
	117	0.82	1.181		13	2118			
	179	0.81	1.274		13	2120			
12	8	0.79	0.653	0.883	13	515	5.8858	0.0942	1.60 %
	126	0.80	0.750		15	633			
	29	0.84	0.860		21	842			
	86	0.82	0.943		21	842			
	91	0.85	0.944		23	941			
	165	0.81	1.050		22	894			
	110	0.79	1.062		20	816			
13	3	0.82	0.610	0.837	13	532	5.8606	0.0910	1.55 %
	43	0.86	0.674		18	696			
	60	0.79	0.760		15	614			
	106	0.82	0.811		18	697			
	176	0.83	0.888		20	788			
	9	0.80	1.130		22	900			
	28	0.81	1.131		23	941			

Table 5, Cont. 3.

Test series	Specimen ¹⁾ number	Sheet thickness (mm)	$\sum \frac{n}{N}$	$\overline{\sum \frac{n}{N}}$ ²⁾	Total life ³⁾ in periods	N' = total life in cycles (kc)	$\overline{\log N'}$	$\sigma \log N'$	Coefficient of variation of $\log N'$
14	75	0.80	0.710	0.840	14	560	5.8493	0.0775	1.33 %
	122	0.83	0.772		17	674			
	93	0.84	0.779		18	735			
	132	0.78	0.795		14	572			
	10	0.83	0.846		19	775			
	61	0.82	0.898		19	776			
	169	0.80	1.149		23	923			
15	44	0.83	0.353	0.552	5	1549	6.3176	0.1150	1.82 %
	64	0.81	0.488		5	1632			
	42	0.79	0.535		6	1819			
	32	0.82	0.557		7	2201			
	38	0.80	0.583		6	1953			
	67	0.85	0.626		8	2604			
	57	0.83	0.834		10	3250			
17	98	0.81	1.242	2.715	14	1141	6.4156	0.3314	5.17 %
	162	0.88	1.390		22	1793			
	74	0.83	1.756		22	1793			
	27	0.81	1.774		20	1630			
	134	0.78	3.306		33	2617			
	92	0.80	5.665		61	4972			
	178	0.82	10.798		128	10432			
18	139	0.82	0.554	0.759	3	487	5.8182	-	-
	16	0.81	0.672		4	627			
	89	0.84	0.886		6	830			
	35	0.81	1.004		6	930			

- 1) The specimens have been arranged in order of increasing life.
- 2) $\overline{\sum \frac{n}{N}}$ is the logarithmic mean of $\sum \frac{n}{N}$.
- 3) The life in periods has been rounded to the nearest integer.

Table 6. Test results of programm tests on 2024-specimens.

Test series	Specimen ¹⁾ number	Sheet thickness (mm)	$\sum \frac{n}{N}$	$\overline{\sum \frac{n}{N}}$ ²⁾	Total life in periods ³⁾	N' = total life in cycles (kc)	$\overline{\log N'}$	$\sigma \log N'$	Coefficient of variation of log N'
21	113	0.77	2.065	2.903	20	8649	7.1159	0.0985	1.38 %
	167	0.78	2.747		28	12108			
	162	0.79	2.903		31	13405			
	65	0.76	2.918		27	11676			
	97	0.81	3.121		37	15983			
	85	0.78	3.336		34	14703			
	161	0.80	3.475		39	16863			
22	73	0.80	1.241	1.561	14	6057	6.8415	0.0675	0.99 %
	131	0.78	1.248		13	5606			
	147	0.79	1.428		15	6502			
	66	0.76	1.470		14	5688			
	107	0.81	1.512		18	7464			
	87	0.78	1.790		18	7794			
	126	0.77	2.563		25	10638			
23	40	0.79	1.468	2.133	16	6681	6.9909	0.1543	2.21 %
	179	0.85	1.609		23	9692			
	181	0.80	1.669		19	7972			
	14	0.78	1.832		19	7972			
	105	0.77	2.139		21	8834			
	121	0.79	2.673		29	12270			
	78	0.76	4.864		45	19370			
24	145	0.81	0.813	1.305	10	4313	6.7875	0.1140	1.68 %
	128	0.77	1.056		11	4691			
	67	0.76	1.317		13	5555			
	138	0.78	1.335		14	6049			
	154	0.79	1.537		17	7339			
	90	0.78	1.556		17	7236			
	77	0.80	1.782		21	9013			

Table 6 . Cont. 1.

Test series	Specimen ¹⁾ number	Sheet thickness (mm)	$\sum \frac{n}{N}$	$\overline{\sum \frac{n}{N}}$ ²⁾	Total life ³⁾ in periods	N' = total life in cycles (kc)	$\overline{\log N'}$	$\overline{\sigma \log N'}$	Coefficient of variation of log N'
25	163	0.78	1.316	1.955	14	11575	7.2222	0.1162	1.61 %
	109	0.80	1.340		15	12517			
	70	0.76	1.840		18	14587			
	101	0.77	2.088		21	17411			
	115	0.79	2.248		24	20026			
	10	0.78	2.649		27	22530			
	170	0.85	2.709		38	31709			
27	172	0.82	0.688	1.100	9	3872	6.7222	0.0759	1.13 %
	100	0.79	0.956		11	4693			
	82	0.82	1.041		13	5619			
	8	0.77	1.108		11	4755			
	180	0.76	1.158		11	4755			
	27	0.79	1.282		15	6328			
	48	0.78	1.734		18	7784			
28	62	0.78	6.184	7.761	63	27243	7.5389	0.0831	1.10 %
	69	0.77	7.344		71	30703			
	47	0.81	7.671		90	38919			
	134	0.74	7.726		65	28108			
	2	0.79	7.961		85	36757			
	104	0.79	8.429		90	38919			
	99	0.80	9.385		105	45413			

1) The specimens have been arranged in order of increasing life.

2) $\overline{\sum \frac{n}{N}}$ is the logarithmic mean of $\sum \frac{n}{N}$.

3) The life in periods has been rounded to the nearest integer.

fig. 1a: Loadspectrum =
= sumdistribution function of the
load.

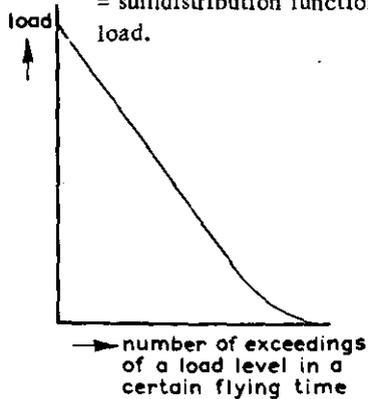


fig. 1b: Approximation of the
loadspectrum by a number of
discrete load levels

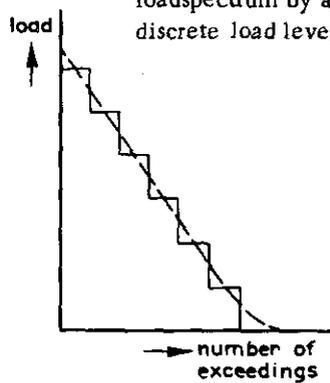


fig. 1c: Histogram of loadings

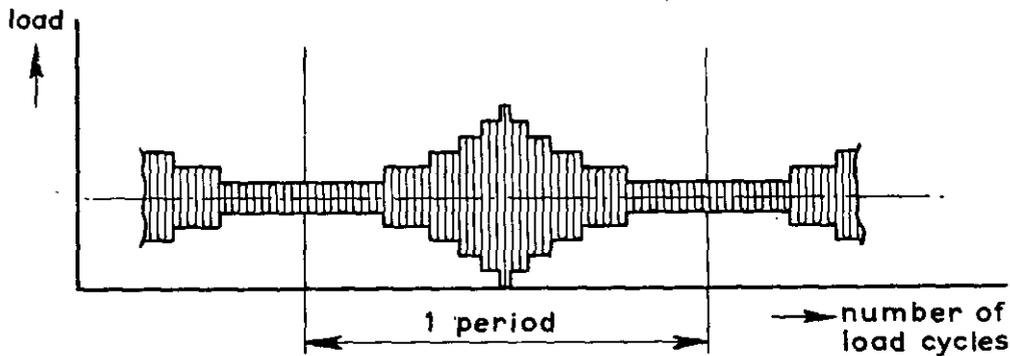
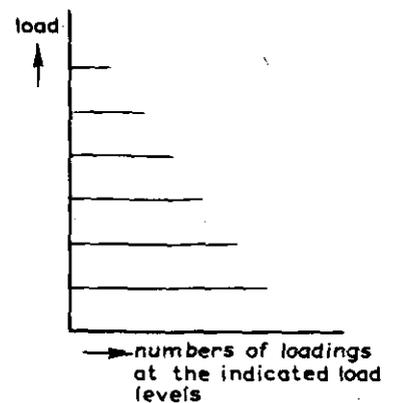
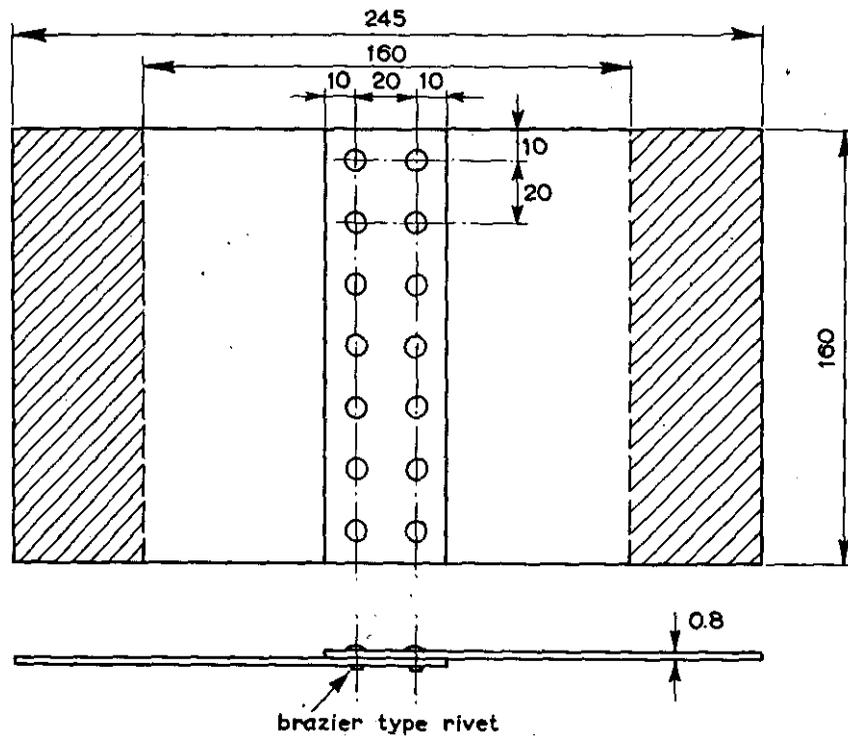


fig. 1d: Loadsequence in a program fatigue test :

The load levels and the number of load cycles per period are in accordance with fig. 1c.
The loadsequence of a period is repeated periodically until failure.

- Parameters to be selected are :
- (1) Number of load levels
 - (2) Magnitude of load levels
 - (3) Highest load level to be included
 - (4) Lowest load level to be included
 - (5) Number of load cycles in one period = period length
 - (6) Sequence of load levels in one period. In fig. 1d an increasing-decreasing order is shown. Other usual sequences are :
increasing order only, decreasing order only and a random sequence.

Fig. 1. The loadspectrum, the load histogram and the program-fatigue test and its parameters.



Dimensions in mm (1" = 25.4 mm)
 Sheet material : 2024 Alclad or 7075 clad

Rivet material : 2117

Fig. 2. Dimensions of the specimens

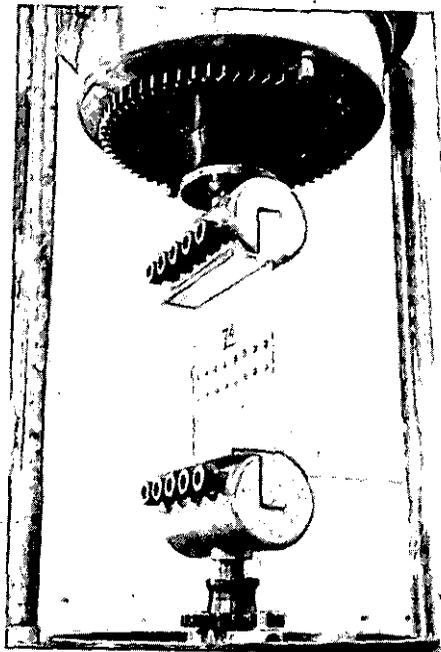


Fig. 3. Specimen with specially designed clamping heads.

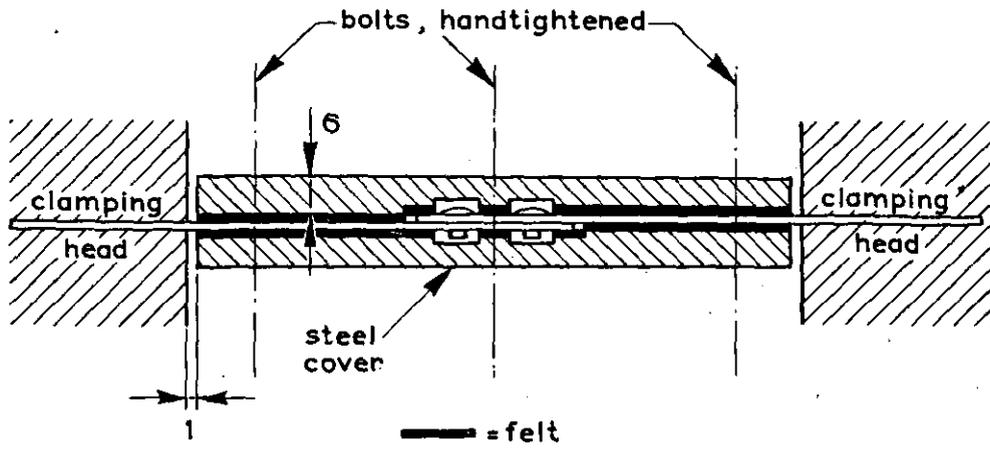


Fig. 4. Anti-buckling guides for compressive loads.

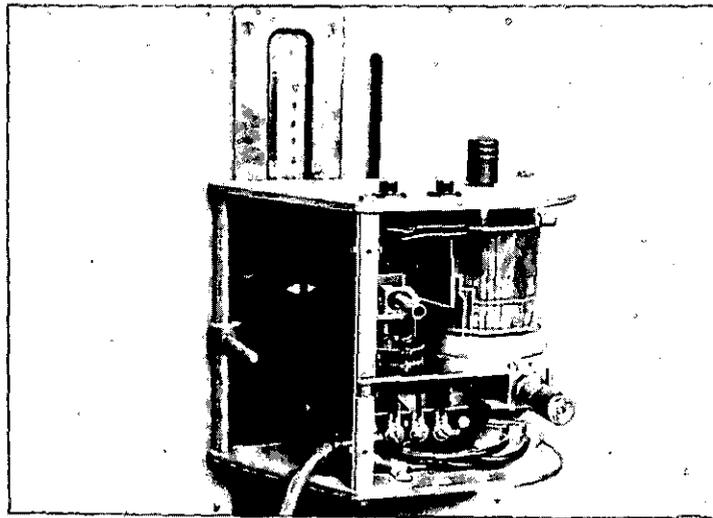
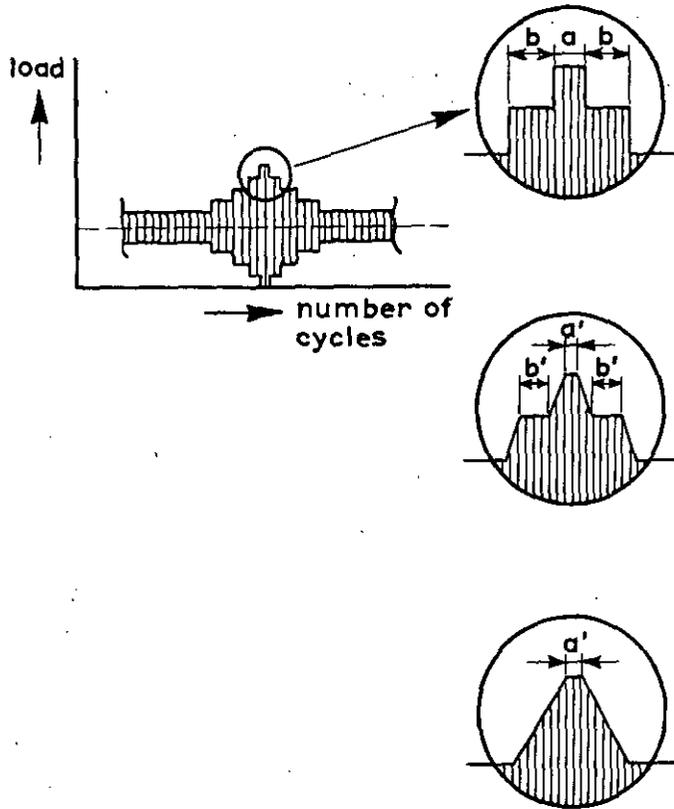


Fig. 5. Automatic program apparatus, especially developed for the Amsler Vibraphore.



I : Desired load sequence.

II : Possible load sequence. For some testseries a' (and b') would have been too large if fully automatic load control had been applied. Then manual control of the program apparatus was necessary.

III : This simplified load sequence was applied instead of II if b' was small in comparison with the time intervals involved in changing load levels. The determination of a' was done in such a way that the same calculated damage as in case I was obtained.

Fig. 6. Approximation of the sequence of the highest load amplitude(s), applied in some testseries.

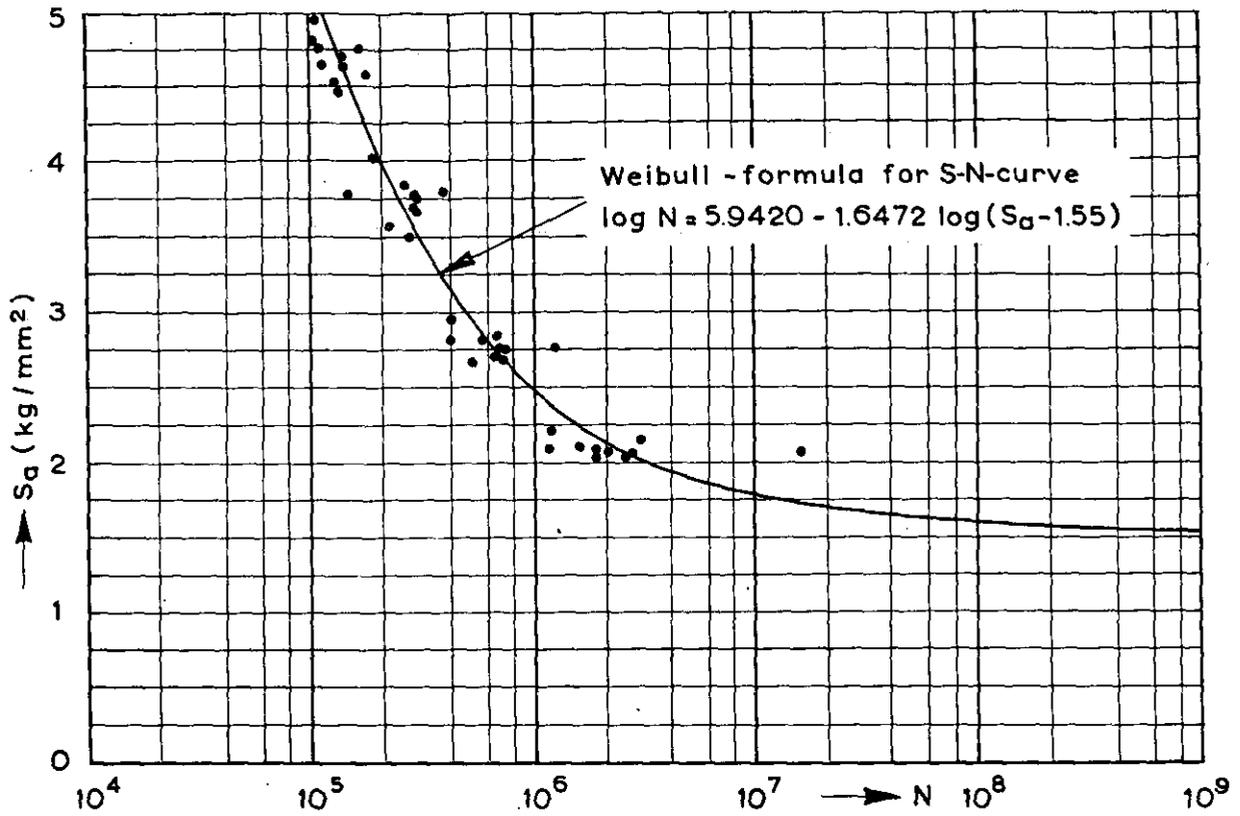


Fig. 7. Fatigue test results for 7075 specimens, tested at $S_m = 6.30 \text{ kg/mm}^2$

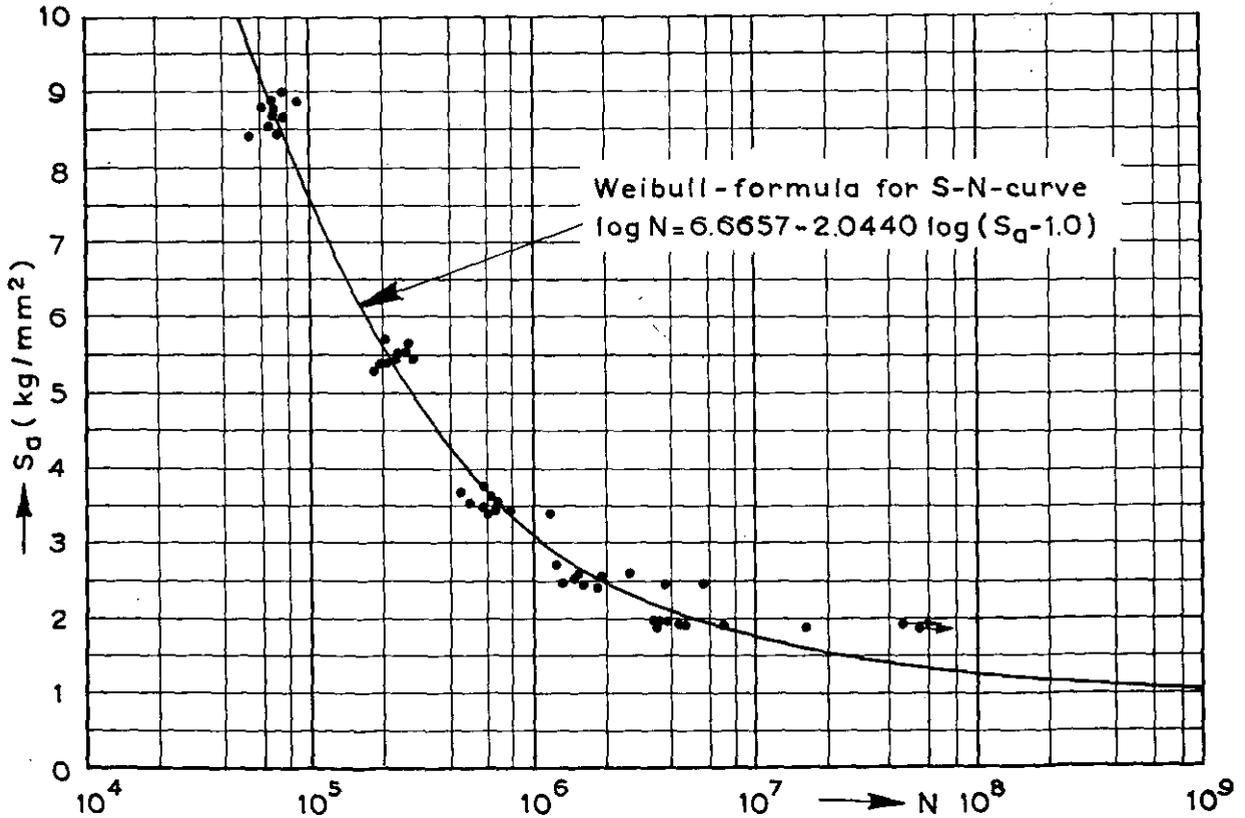


Fig. 8. Fatigue test results for 2024 specimens, tested at $S_m = 9.00 \text{ kg/mm}^2$

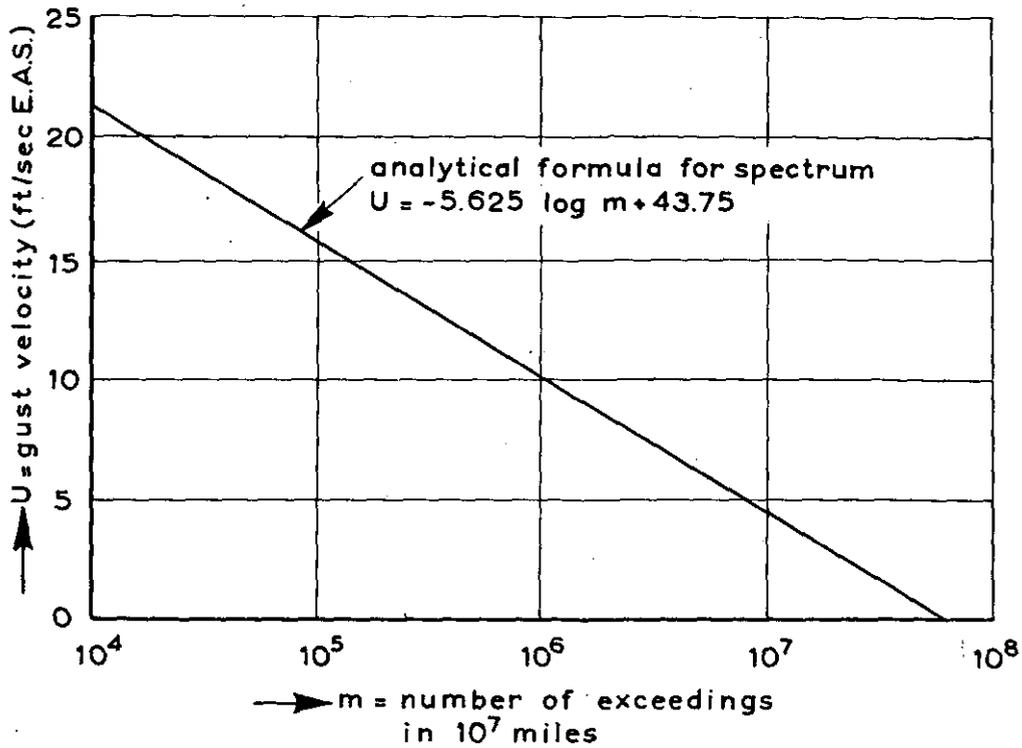
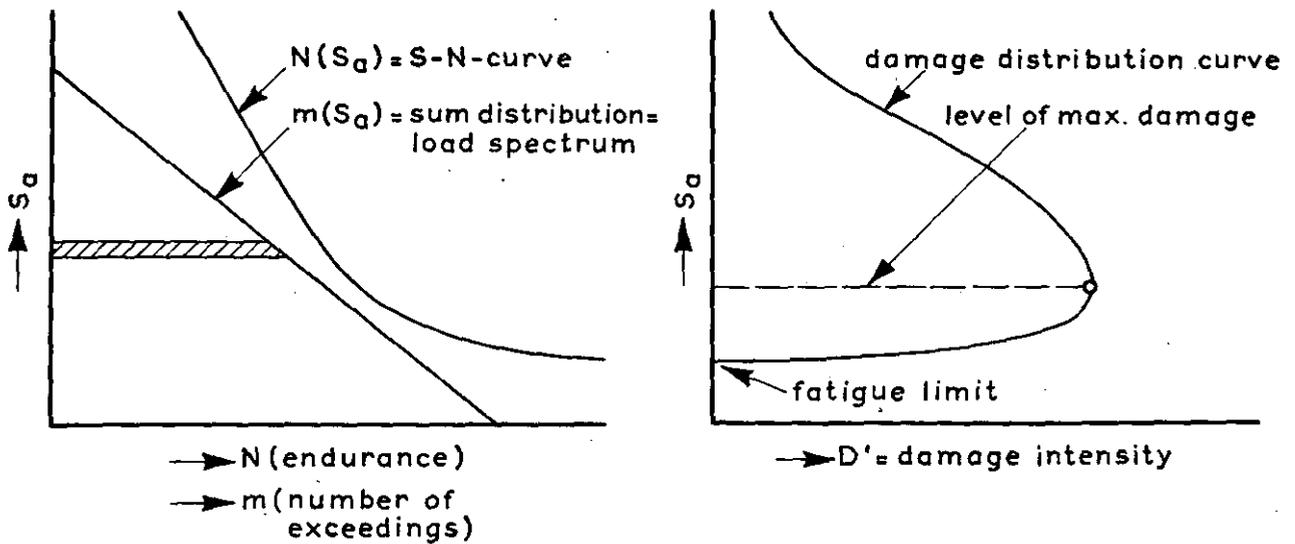


Fig. 9. Gust spectrum of Taylor at a height of ca. 7000 ft for 10^7 miles (ref. 51).



Number of loadcycles in hatched interval : $m(S_a) - m(S_a + dS_a) = -dm$

The corresponding damage increment : $dD = -\frac{dm}{N}$

Damage intensity $D' = \frac{dD}{dS_a} = -\frac{1}{N} \cdot \frac{dm}{dS_a}$

Damage-distribution curve = D' as a function of S_a .

Total damage due to all S_a -values = $D = \int dD = \int D' dS_a =$ area under the damage-distribution curve.

Fig. 10. The construction of the damage-distribution curve, based on the Palmgren-Miner rule.

Method reported by Chilver (ref. 6).

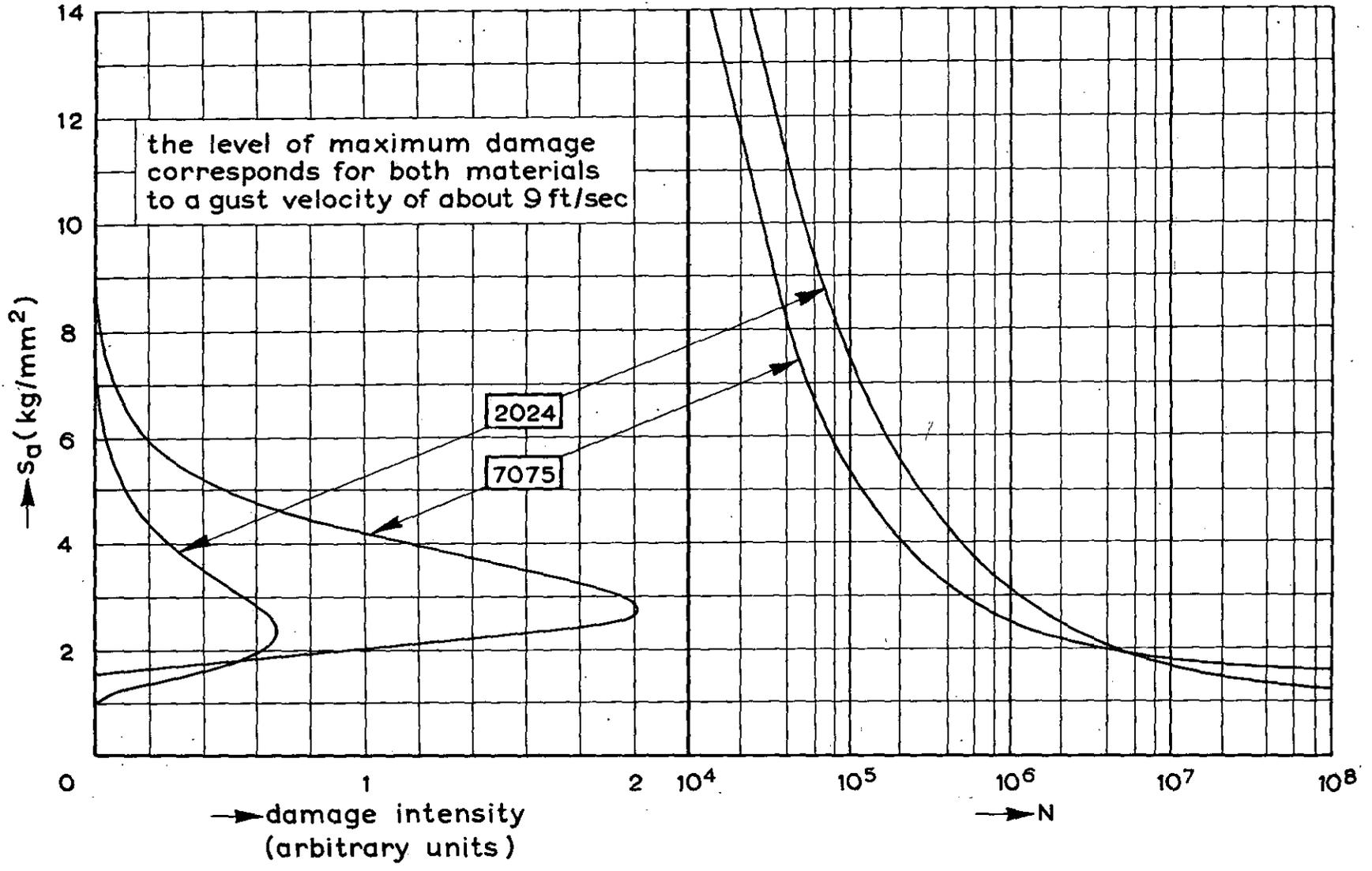


Fig. 11. Fatigue curves and damage distribution curves for 7075 and 2024 specimen.

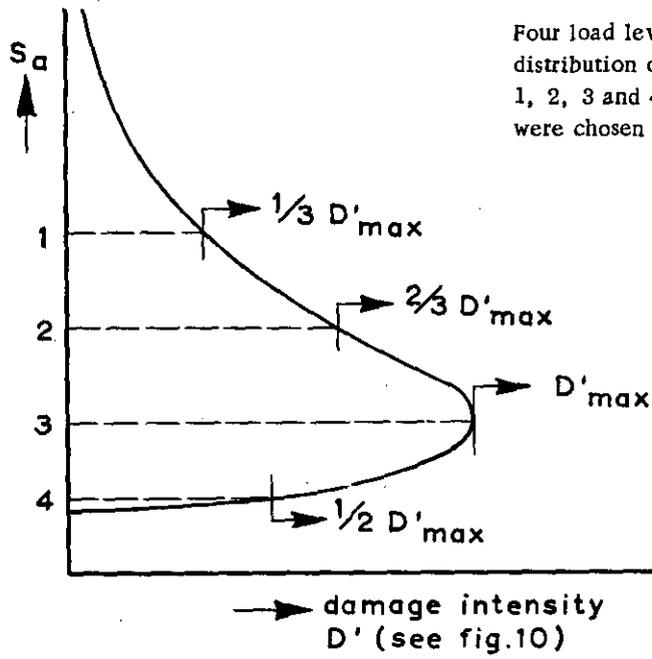


Fig. 12. Determination of the stress amplitudes in the program-tests.

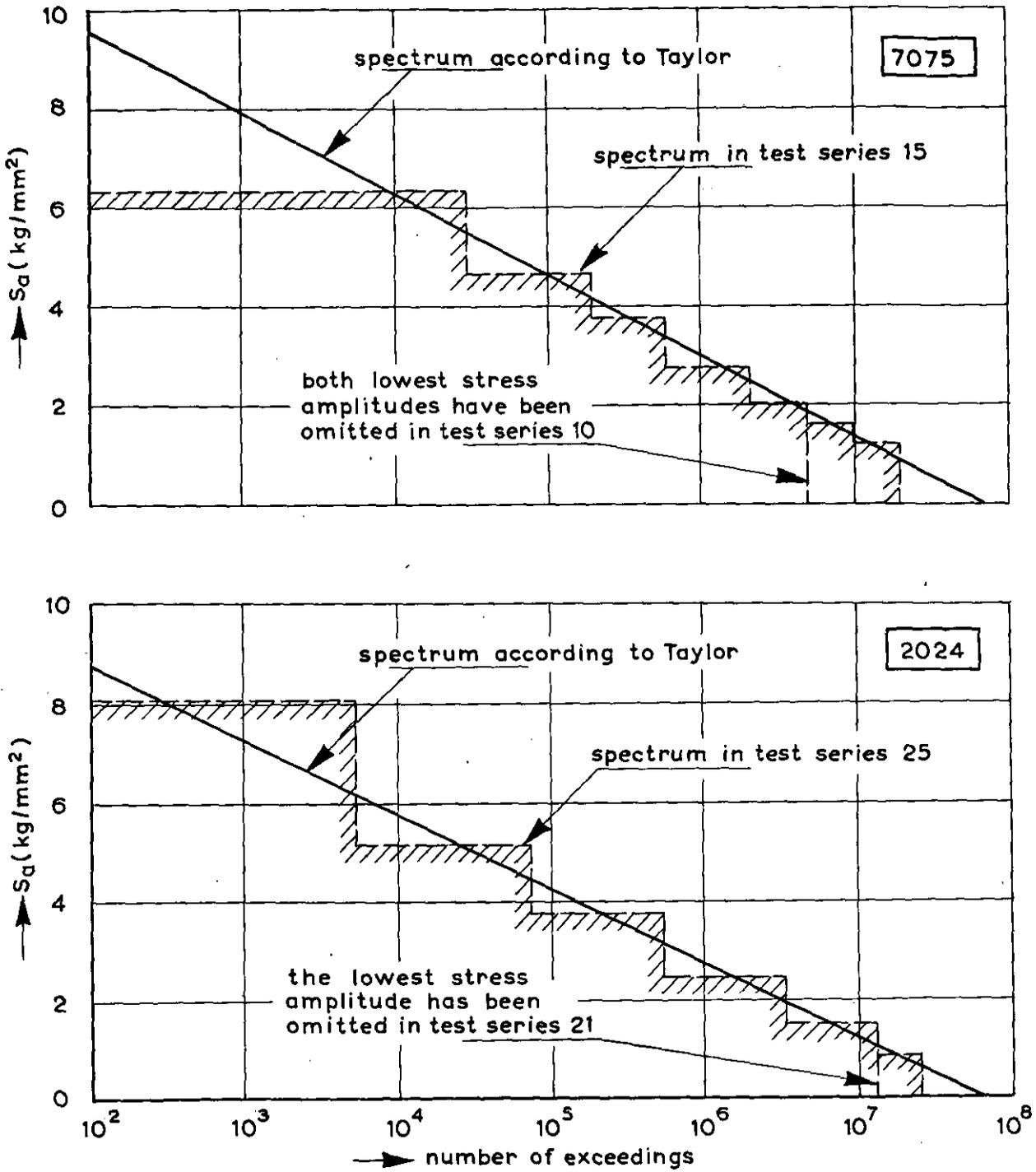


Fig. 13. Stress amplitude spectrum according to Taylor and stepped spectrum in program tests.

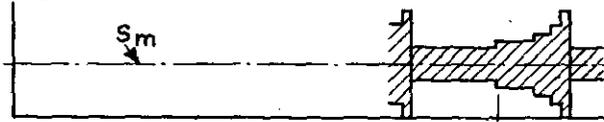
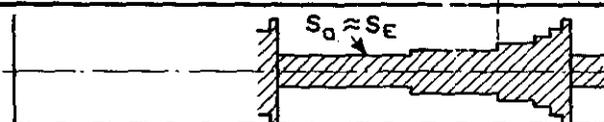
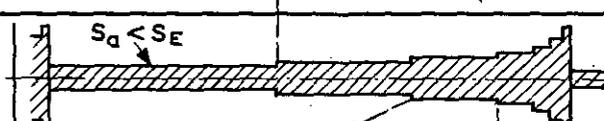
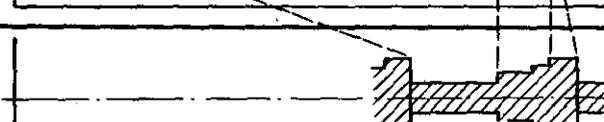
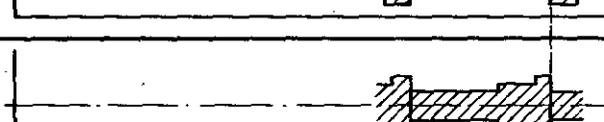
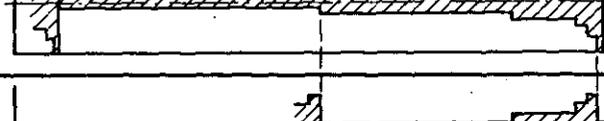
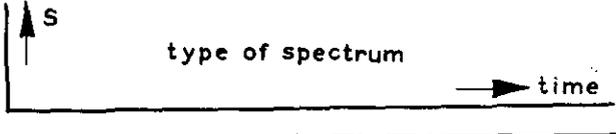
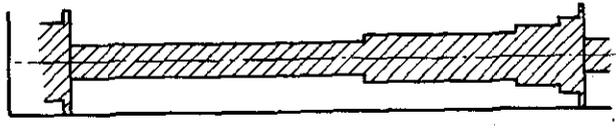
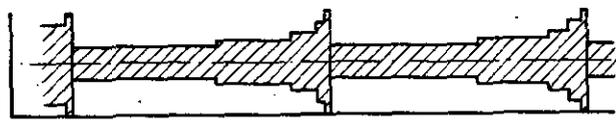
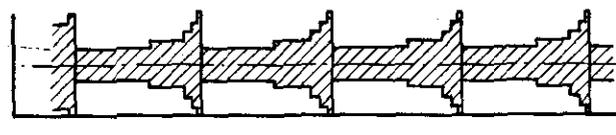
Material	Test series	 type of spectrum	Similarity with other test series	$\frac{\sum N}{N}$
7075	10	 S_m	Taylor's gust spectrum, see Fig. 13	1.08
	11	 $S_a \approx S_E$	Similar to series 10, one lower S_a added	0.91
	15	 $S_a < S_E$	Similar to series 10, two lower S_a 's added see Fig. 13	0.55
	4		Similar to series 10, highest S_a omitted and lowest S_a extended three times	0.59
	3		Similar to series 10, highest S_a omitted and the highest S_a but one extended three times	0.87
	2		Similar to series 10, two highest S_a 's omitted	0.71
7075	18		Taylor's gust spectrum. Similar to series 10, period length increased twice	0.76
	1		Similar to series 18, highest S_a omitted	0.76
2024	21		Taylor's gust spectrum, see Fig. 13	2.90
	25	 $S_a < S_E$	Similar to series 21, one lower S_a added, see Fig. 13	1.96
	24		Similar to series 21, highest S_a omitted	1.31

Fig. 14. The effect of the type of spectrum on the endurance.

Material	Testseries	type of spectrum	Similarity with other testseries	$\sum \frac{1}{N}$	Mean life in period
7075	12		Taylor's gust spectrum increasing S_a	0.88	19
	13		Similar to series 13, however, decreasing S_a	0.84	18
	14		Similar to series 12 and 13, however, increasing S_a and decreasing S_a , period length twice as large	0.84	9
	10		Similar to series 14, however, increasing S_a	1.08	12
	21		Taylor's gust spectrum, increasing S_a	2.90	31
	22		Similar to series 21, however, decreasing S_a	1.56	17
2024	23		Similar to series 21 and 22, however, increasing and decreasing S_a	2.13	25

Fig. 15. The effect of the order of succession of the load-steps on the endurance under program-fatigue loading.

material	testseries		Similarity with other testseries	1) $\frac{\sum n}{N}$	Mean life in periods
7075	18		Taylor's gust spectrum	0.76	5
	10		Similar to series 10, period length reduced twice	1.08	12
	12		Similar to series 10, period length reduced 4 x	0.88	19

1) The probability that the results of testseries 18 and 10 are different is about 93% and the probability that the results of testseries 10 and 12 are different is about 90%.

Fig. 16. The effect of the length of the period on the endurance under program-fatigue loading.

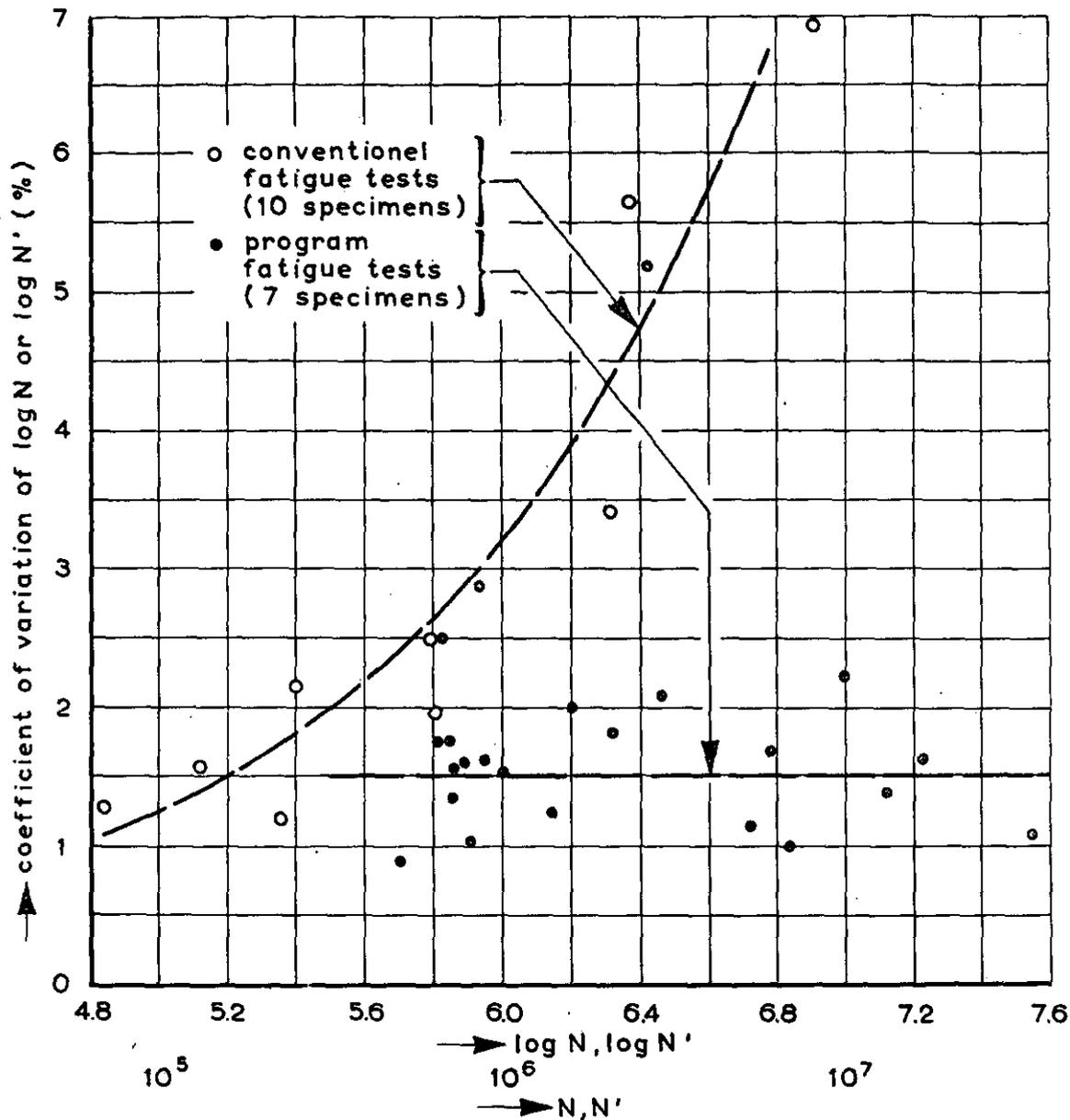
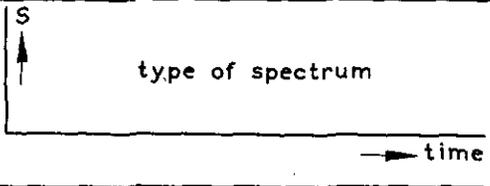
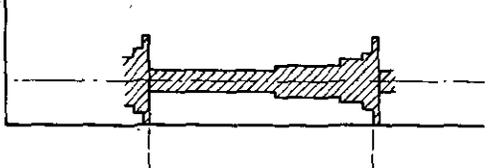
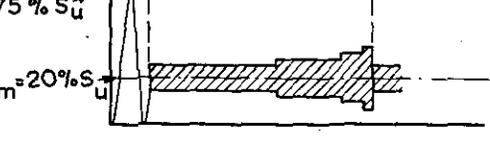
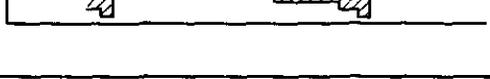
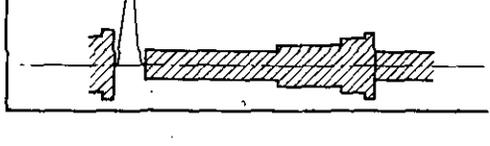
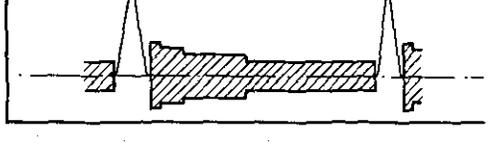
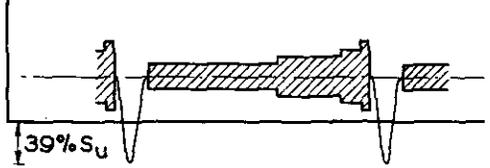


Fig. 18. Comparison of the scatter in one load level fatigue tests and the scatter in program-fatigue tests.

Material	Test series		Similarity with other test series	$\frac{\sum n}{N}$
7075	10		Taylor's gust spectrum	1.08
	5		One high pre-load, spectrum of series 10, highest S_a omitted	1.42
	6		Spectrum of series 5, one high positive load at the end of each period	9.90
	6a		Similar to series 6, however high loads omitted after $\sum \frac{n}{N} \sim 4.65$ (50th period)	5.51
	6b		Similar to series 6, however high loads at the end of odd numbered periods only	3.68
	17		Similar to series 6, however, spectrum in reversed order	2.72
	7		Similar to series 6, however, high negative loads instead of high positive loads	0.51

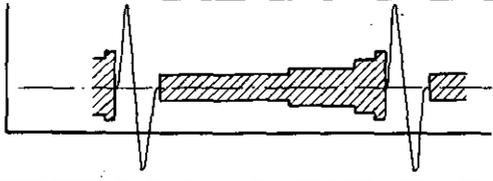
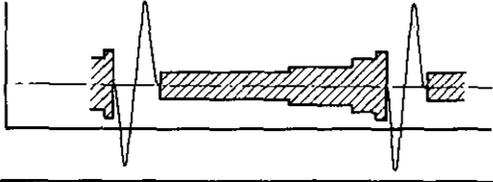
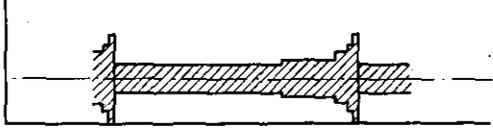
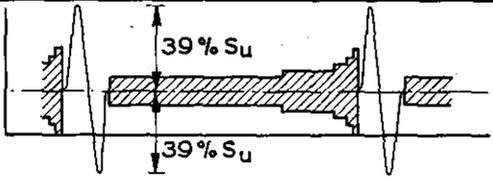
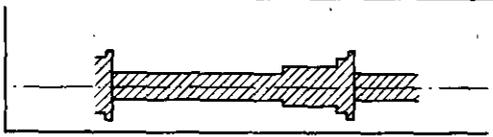
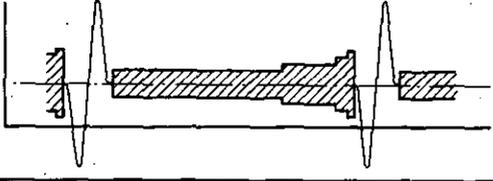
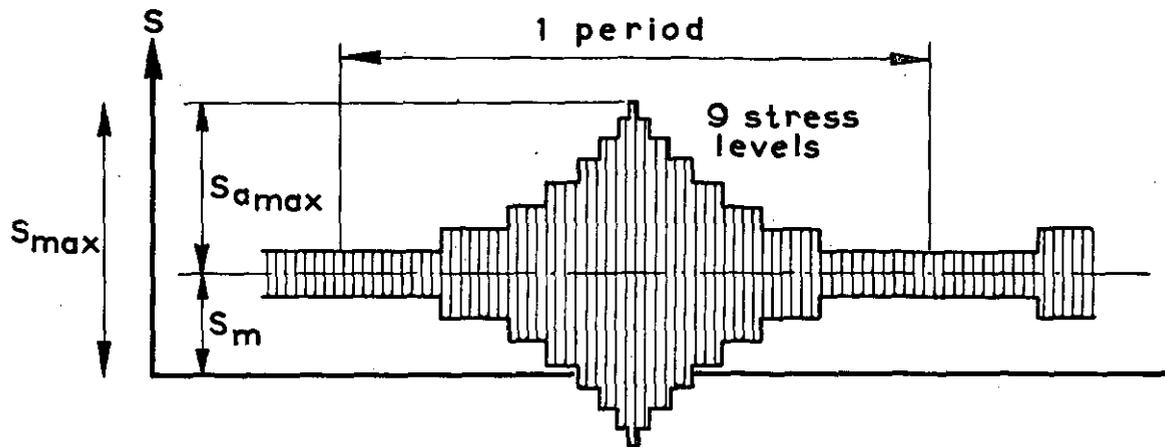
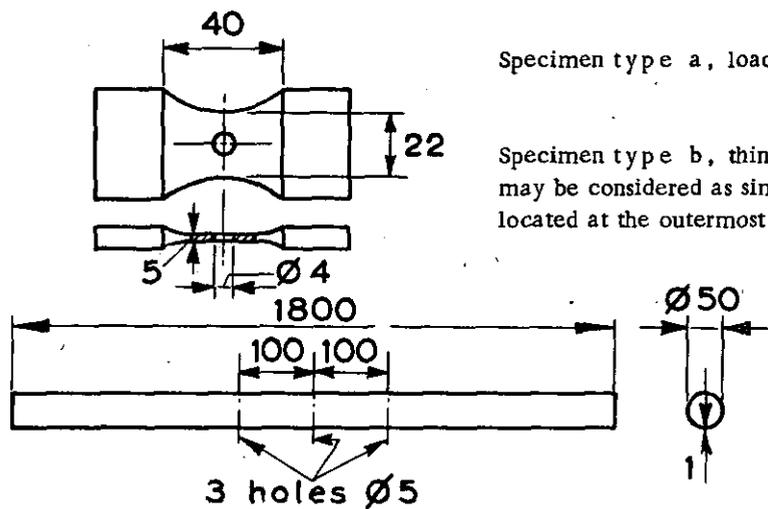
	8		Similar to series 6, high positive loads now followed by high negative loads	0.70
	9		Similar to series 8, high loads in reversed order	3.11
2024	21		Taylor's gust spectrum	2.90
	27		Similar to series 21, one high positive and negative load at the end of each period	1.10
	24		Taylor's gust spectrum, similar to series 21, highest S_a omitted	1.31
	28		Similar to series 24, one high negative and positive load at the end of each period	7.75

Fig. 17. The effect of several types of high loads on the endurance under program-fatigue loading. Period length corresponds to $\sum \frac{n}{N} \sim 0.1$.



Gust spectrum :

$\frac{S_a}{S_{a_{max}}}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{13}{16}$	$\frac{15}{16}$	$\frac{16}{16}$
Number of cycles in one period	703000	164250	28750	3600	1080	328	126	40	2



Specimen type a, loaded in plane bending, $k_t \sim 2.0$

Specimen type b, thin-walled tube loaded in plane bending, which may be considered as simulated axial loading since the notches are located at the outermost fibres, $k_t \sim 2.25$.

Fliegwerkstoff	Type of alloy	$S_{0.2}$ (kg/mm ²)	S_u (kg/mm ²)
3115.4	Al Cu Mg	33.0	45.5
3115.5		32.8	46.5
3125.4		37.0	48.5
3125.5		33.0	49.0
3315.7	Al Mg	26.0	35.0

Materials :

Fig. 19. Program-fatigue tests with a gust spectrum on two types of notched specimens of some aluminum alloys. Tests reported by Gassner (ref.18, additional information provided by private communication).

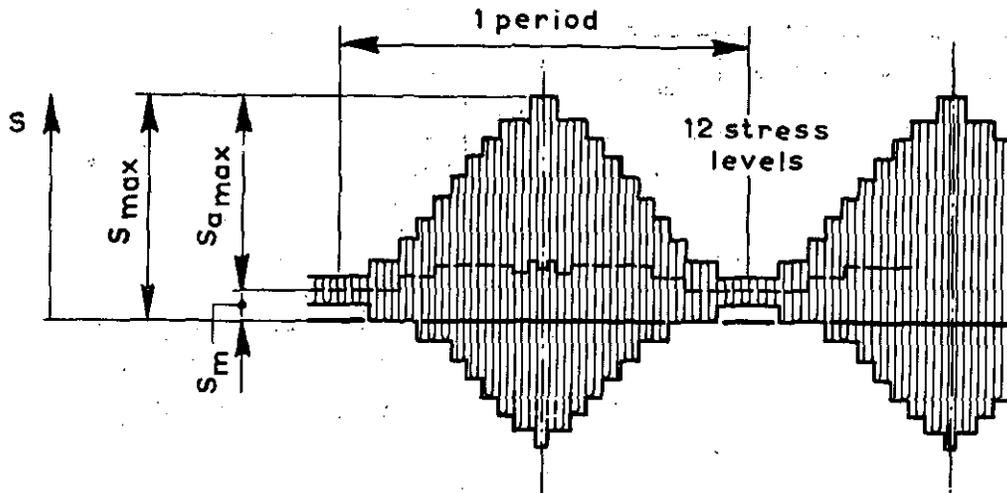
Material	Type of specimen	S_m ¹⁾	S_{max} ¹⁾ (kg/mm ²)	N at S_{max} (kc)	$\sum \frac{n}{N}$ ²⁾	Number of periods
3125.4	a	$\frac{S_{max}}{2.4}$	58.7	<1	0.25	0.6
			53.7	1.6	0.74	3
			49.6	9.5	0.72	5
			49.1	10.3	1.02	8
			46.3	13.8	1.06	12
			41.0	24	0.61	20
		$\frac{S_{max}}{3.4}$	52.5	1.2	1.14 ³⁾	3
			50.8	1.7	0.50-0.80	2-3
			50.8	1.7	1.59 ³⁾	6
			47.7	3.0	2.13 ³⁾	9
			47.0	3.4	1.58 ³⁾	9
			45.0	4.6	0.96	5
			44.5	5.1	2.20 ³⁾	13
			42.5	6.8	1.23-1.38	0-10
			38.3	12.2	0.89	15
			35.1	18.2	0.77	24
		31.0	29.0	0.65	35	
		0	43.5	<1	0.56	2
			40.0	1.6	0.94	5
			35.7	3.5	1.03	9
			32.9	5.9	1.20	15
3125.5	b	$\frac{S_{max}}{3.4}$	29.5	1.1	1.64	2
			29.3	1.2	0.86	1
			25.0	3.7	1.63	5
			22.5	7.4	2.00	12
			17.0	31.8	1.53	30
			16.5	35.8	1.85	40
			13.0	87.0	0.30	40
3115.4	a	$\frac{S_{max}}{3.4}$	51.0	1.0	0.63-0.63	3-3
			45.5	3.8	1.69	8
			45.0	4.4	1.20	7
			42.0	6.0	1.46-1.46	10-10
			38.0	13.2	1.83	24
			35.0	19.5	0.97	30
3115.5	b	$\frac{S_{max}}{3.4}$	25.7	3.5	1.00 ⁴⁾	3
			23.7	5.8	3.88-3.89 ⁴⁾	17-17
			23.5	6.0	1.48-1.48	7-7
			19.0	19.7	2.53 ⁴⁾	27
			19.0	19.7	1.16-1.40	14-18
			16.5	37.0	1.73	47
			13.0	95.0	0.39	47
3315.7	b	$\frac{S_{max}}{3.4}$	22.8	1.2	1.06	3
			19.1	3.7	1.84	10
			17.0	7.2	1.44-2.08	15-22
			15.2	12.8	0.91-1.25	19-25

1) Nominal bending stresses.

2) Individual test results.

3) Lowest stress amplitude omitted.

4) Tests with rest periods of 48 hours after each program-period.



Spectrum: combined gust and maneuver spectrum.

$S_a/S_{a \max}$	0.771	0.736	0.683	0.654	0.570	0.491	0.406	0.314	0.279	0.181	0.114	0.053
$S_m/S_{a \max}$	0.23	0.26	0.21	0.24	0.24	0.24	0.24	0.24	0.20	0.20	0.14	0.14
Number of cycles in one period	4	20	64	88	164	260	400	600	600	1400	12,400	316,700

Specimen: Type a (Flw. 3125.4) and b (Flw. 3115.5), see fig. 19

Test results

Material	Specimen (see fig. 19)	$S_{a \max}$	$N_{S_{a \max}}$	$S_{a \min}$	$N_{S_{a \min}}$	$\sum \frac{n}{N}$
3125.4	Type a loaded in bending	42.6	600	2.9	8	0.63
		39.6	1200	2.7	8	0.63
		39.6	1200	2.7	8	0.76
		35.8	2800	2.5	8	0.59
		35.8	2800	2.5	8	0.87
		32.4	5100	2.2	8	0.74
3115.5	Type b simulated axial loading	27.0	200	1.9	8	1.78
		23.1	700	1.6	8	1.41
		23.1	700	1.6	8	1.61
		19.7	2200	1.4	8	0.70
		19.7	2200	1.4	8	0.77
		19.7	2200	1.4	8	0.85
		17.0	5800	1.2	8	0.94

Fig. 20 Program-fatigue tests with a combined gust and maneuver spectrum on two types of notched specimens of aluminum alloy (Tests reported by Gassner, ref. 18).

Specimen : Specimen with a central hole, type a of fig. 19.

Material : Flw. 3125.4 (AlCuMg), properties see fig. 19.

Loadsequence : Program loading with 9 stress levels, see fig. 19.

Positive mean stress $S_m = \frac{S_{\max}}{3.4}$ (S_{\max} = maximum stress of program)

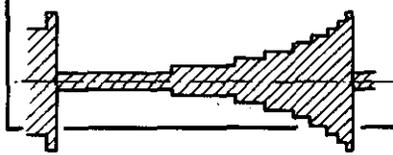
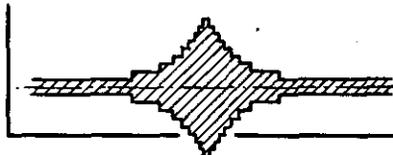
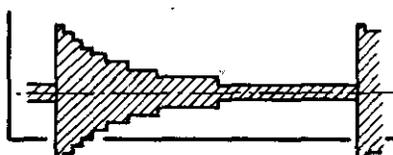
Test results :

Loadcycles per period	Number of periods until failure	S_{\max} (kg/mm ²)	N at S_{\max}	$\sum \frac{n}{N}$ ¹⁾
5 400 000	1	42	7500	0.70
900 000	6	43	6500	0.79
360 000	15	46	4000	1.11
180 000	30	45	4600	1.02

¹⁾ Values for $\sum \frac{n}{N}$ were obtained by interpolation from test results. This interpolation was necessary since Gassner wanted to compare different values of the period length at the same total life of 5 400 000 cycles. Consequently he found slightly different values for S_{\max} (3 rd column).

Fig. 21 The effect of the length of the period on the endurance under program loading, Results reported by Gassner (ref. 18).

For the number of load cycles in one period at the nine stresslevels, see fig. 19. The material was Flw. 3125. 4 (Al Cu Mg) and the specimen was type a, both being also indicated in fig. 19. In all tests the number of cycles in one period was 900 000 and $S_{\max} / S_m = 3.4$.

LOADSEQUENCE	Note	S_{\max} (kg/mm ²)	N at S_{\max}	$\sum \frac{n}{N}$ ¹⁾
	increasing amplitudes	45	4600	1.02
	increasing and decreasing amplitudes	43	6500	0.79
	decreasing amplitudes	42	7500	0.70

1) $\sum \frac{n}{N}$ calculations were based on interpolated test results.

Interpolation was necessary since Gassner wanted to compare different load sequences at the same total life of 5 400 000 cycles (=6 periods) and thus he found slightly different allowable maximum stresses (3rd column).

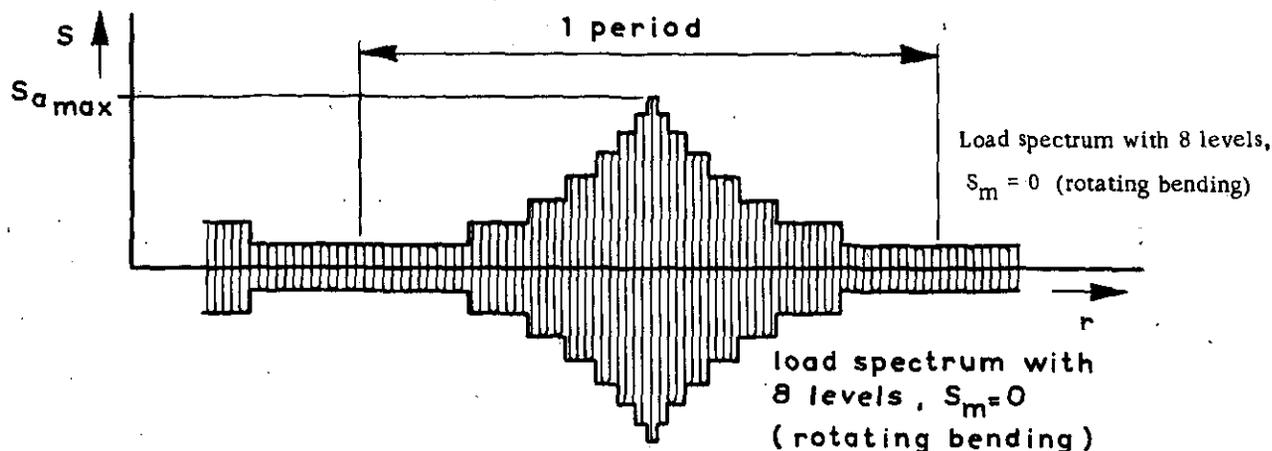
Fig. 22. Comparison of different load sequences in program tests on notched specimens.

Results of Gassner (ref.18)

Spectrum	$S_{a_{\max}}$ ¹⁾ (kg/mm ²)	N at $S_{a_{\max}}$	$S_{a_{\min}}$ ¹⁾ (kg/mm ²)	N at $S_{a_{\min}}$	$\overline{\sum \frac{n}{n}}$ ²⁾	Number of tests
Log binomial spectrum (gust spectrum)	42.1	1250	4.2	∞	0.72	5
	37.0	2700	3.7	∞	0.91	11
	34.0	3750	3.4	∞	0.94	11
	31.2	6100	3.1	∞	0.91	10
	28.0	11000	2.8	∞	0.84	5
Binomial spectrum (maneuver spectrum)	35.0	3400	4.4	∞	1.17	11
	33.3	4300	4.2	∞	1.05	7
	29.5	8100	3.7	∞	1.18	11
	25.0	20000	3.1	∞	1.19	9

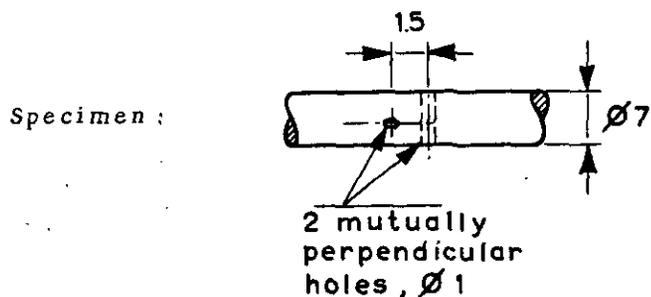
1) Nominal stresses

2) Logarithmic average



Log binomial spectrum= gust spectrum	$S_a/S_{a \max}$	1	0.87	0.74	0.61	0.47	0.34	0.22	0.10
	Number of cycles in 1 period	18	40	142	590	2910	14,400	97,000	885,000
Binomial spectrum	$S_a/S_{a \max}$	1	0.95	0.85	0.725	0.575	0.425	0.275	0.125
	Number of cycles in 1 period	1	8	140	1362	10000	46000	140000	305000

The binomial spectrum involves relatively more high and less low amplitude cycles than the log binomial (gust) spectrum and may be identified as a maneuver spectrum.



Specimens were loaded in rotating bending. $k_t \sim 2.15$

Material : Flw. 3125, Aluminium alloy, 2024 type (Al Cu Mg)

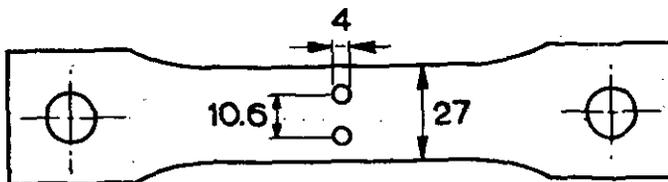
$$S_{0.2} = 37 \text{ kg/mm}^2 (52.6 \text{ k.s.i.}), S_u = 50 \text{ kg/mm}^2 (71.2 \text{ k.s.i.}).$$

Test results

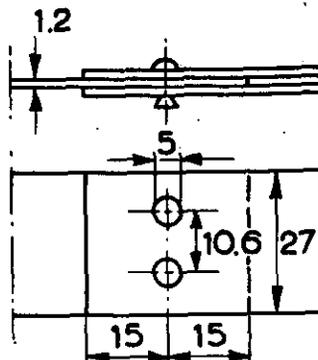
Fig. 23 Program fatigue tests on notched rotating bending specimens of an aluminum alloy, Al Cu Mg-type. Tests reported by Gassner (ref. 21) and additional data from private communication.

Type of specimens:

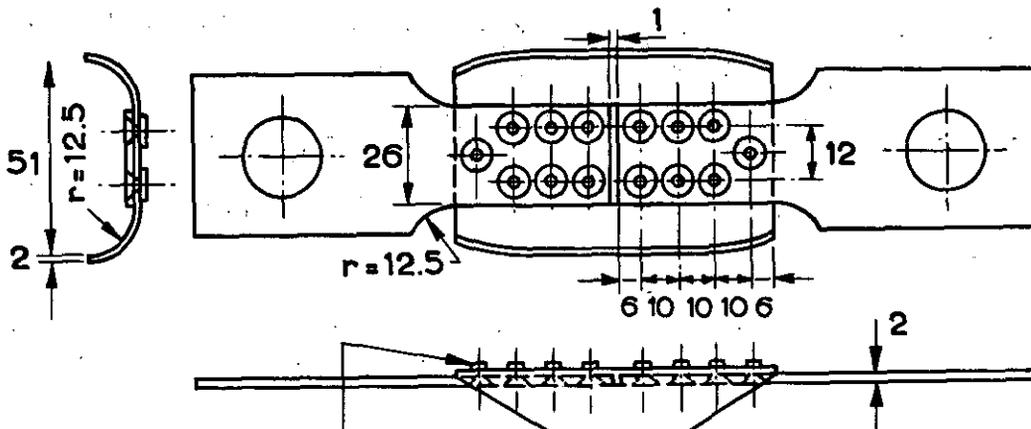
a. specimen with 2 holes



b. double lap joint



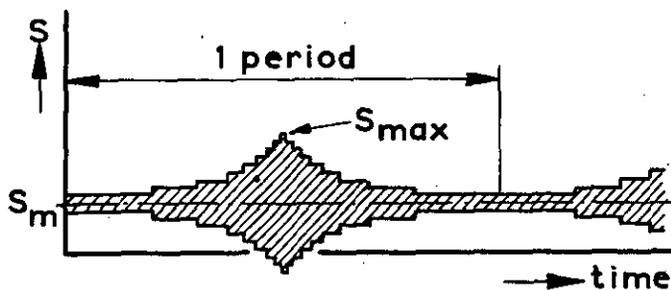
c. riveted single strip joint



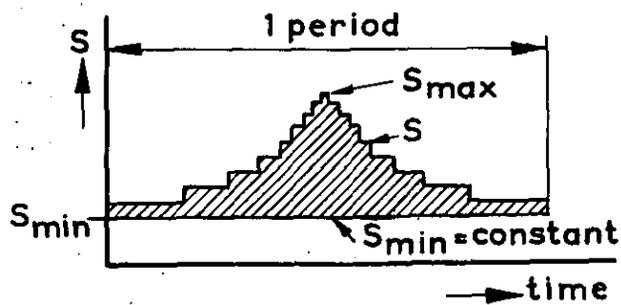
4 mm countersunk rivets

Materials: 2024-T Alclad and 7075-T clad

Loadsequence :



I: gust spectrum



II: maneuver load spectrum

Stress level		1	2	3	4	5	6	7	8	9
Spectrum I	S_a/S_m	0.10	0.29	0.49	0.68	0.88	1.07	1.27	1.46	1.66
	Number of cycles per period	562 000	131 400	23 000	2 880	864	262	100	32	20
Spectrum II	S/S_m	1.4	2.2	3.0	3.8	4.6	5.4	6.2	7.0	7.8
	Number of cycles per period	6 400	3 600	2 200	1 240	700	400	240	136	78

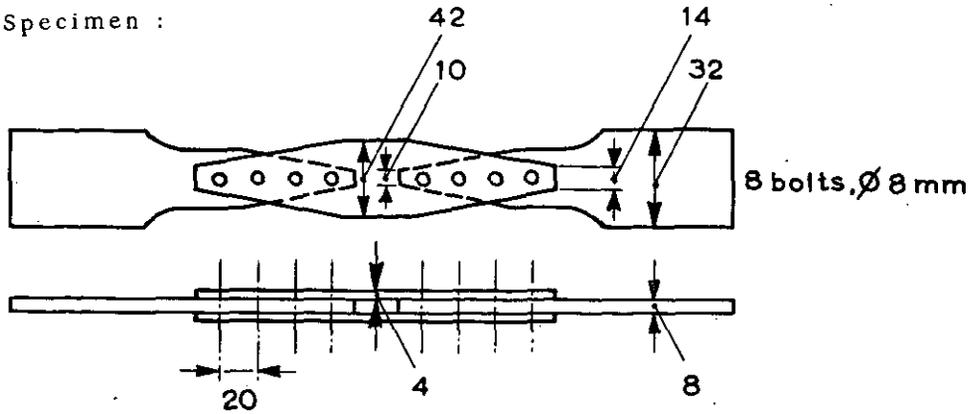
Test results

Type of load spectrum	Type of specimen	Material	S_m (kg/mm ²)	S_{max} (kg/mm ²)	N at S_{max} (10 ³)	Number of stress levels below endurance limit	Number of tests	$\frac{\sum n}{N}$	
I	a	2024	13.0	34.7	1.5	1	3	1.33	
			10.9	29.0	5	2	3	2.28	
			8.7	23.1	16	2	3	3.00	
			6.5	17.3	45	3	3	0.90	
	b		10.9	29.0	3.7	0	2	1.20	
			6.5	17.3	29	1	3	2.33	
			4.3	11.5	120	2	3	0.77	
			3.5	9.3	220	3	2	0.64	
	c	7075	9.3	24.8	1.4	1	4	3.56	
						-	2 ^{*)}	4.70	
			6.2	16.5	14.5	2	2	2.25	
			4.7	12.4	72.0	2	2	1.33	
				-	2 ^{*)}	2.16			
II	b		2024	S_{min} (kg/mm ²)					
				3.6	28.0	6.0	1	2	2.34
				2.7	21.1	16	1	2	1.45
		2.2		16.8	35	2	2	1.20	
		3.73		29.1	0.9	1	2	1.45	
		3.11		24.2	3.1	2	2	1.61	
	2.33	18.2	12.5	3	2	2.51			
	c	7075							

^{*)} Load amplitude below the fatigue limit were omitted

Fig. 24. Program-fatigue tests on notched specimens. Results reported by Wällgren (ref. 53).

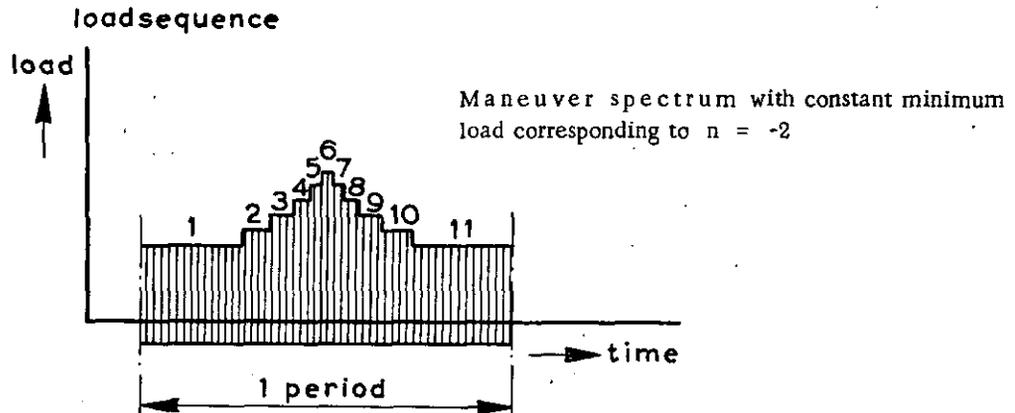
Specimen :



The material was 2024

The ultimate design load was 8400 kg corresponding to a loadfactor $n = 12$

The static failure load was 9180 kg

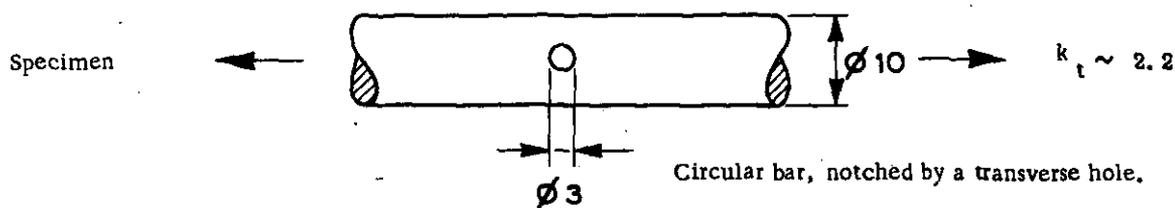


Loadlevel	Loadfactor	Number of cycles per period	Corresponding constant-amplitude endurance (kc)
1-11	2.5	51200	1100
2-10	3.5	9440	600
3-9	4.5	1748	200
4-8	5.5	324	80
5-7	6.5	32	33
6	7.5	8	14

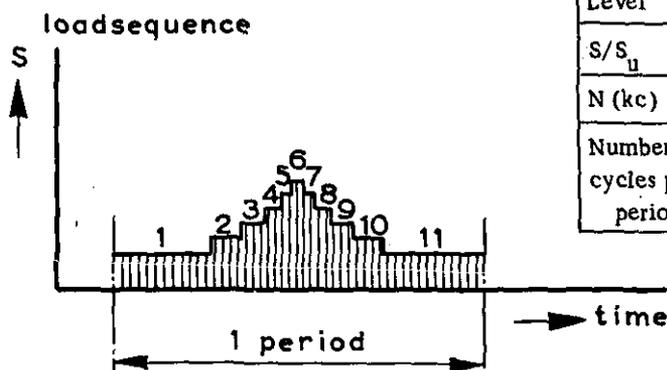
The test result is slightly uncertain since the bolt torque and the bolt hole clearance were not the same for all specimens. Moreover there were two types of failure, viz. one through the bolt holes and one in the tongue at the end of the strip due to fretting. Some tests were not carried on until failure and some tests were started with a slightly modified program. Nevertheless the medium result may be regarded as indicative.

Result of program test : Medium value of 7 tests was 45.5 periods $\cong \sum \frac{1}{N} \sim 3.5$

Fig. 25 Results of program-fatigue tests on a bolted joint. Results reported by Wålgren and Petrelius (ref. 54).



Material 7075-T, extruded material, $S_u = 66 \text{ kg/mm}^2$ for the specimen.



Level	1-11	2-10	3-9	4-8	5-7	6
S/S_u	0.23	0.32	0.41	0.50	0.59	0.68
N (kc)	1250	190	46	15	6	2.7
Number of cycles per period	30000	3000	283	38	9	4

Test results : 9 program-fatigue tests were performed. Numerous constant-amplitude tests had been performed earlier, thus allowing an evaluation on a probability basis. The results were :

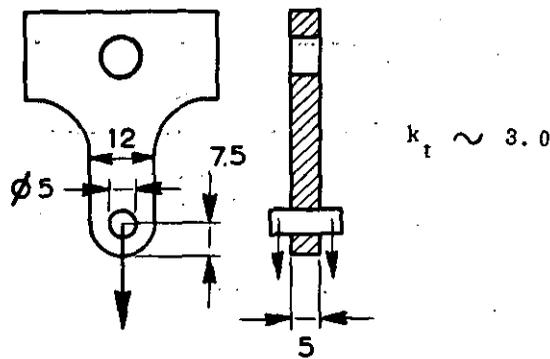
Probability of failure %	Number of periods to failure		$\sum \frac{n}{N}$ ²⁾
	Calculated ¹⁾	Experimental	
10	5.9	34	5.8
50	9.8	40	4.1
90	17.1	46	2.7

1) Indicated values apply to the results of the program-fatigue tests as well as the N-values employed for the calculation of $\sum \frac{n}{N}$.

2) Results published earlier by the authors in ref. 45 differ from the results presented here. The older results were obtained by private communication and not yet corrected for machine errors.

Fig. 26 Results of program-fatigue tests on a notched specimen. Results reported by Wällgren (ref. 55).

Specimen :

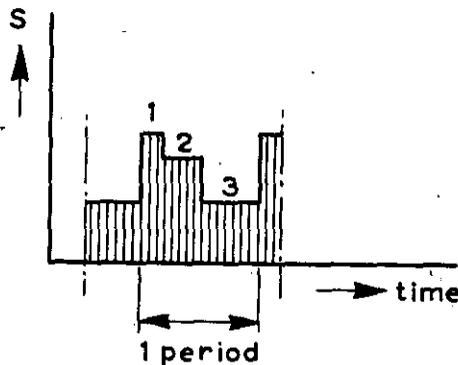


Materials : (1) 2024-type material, extruded. $S_u = 56.7 \text{ kg/mm}^2$,
 $S_{0.2} = 41.8 \text{ kg/mm}^2$, $S_{10} = 14\%$. For the lug
 $S_u = 46.5 \text{ kg/mm}^2$ (net section)

(2) 7075-type material, forged. $S_u = 58.9 \text{ kg/mm}^2$,
 $S_{0.2} = 49.6 \text{ kg/mm}^2$.

S not stated. For the lug $S_u = 49.3 \text{ kg/mm}^2$ (net section)

Loadsequence



Maneuver-type spectrum, $S_{\min} = 0$
 $S_2 = 0.8 S_1$ and $S_3 = 0.5 S_1$

Spectrum	Number of cycles per period for level 1, 2 and 3		
	I	1000	4000
II	1800	7200	9000

Test results

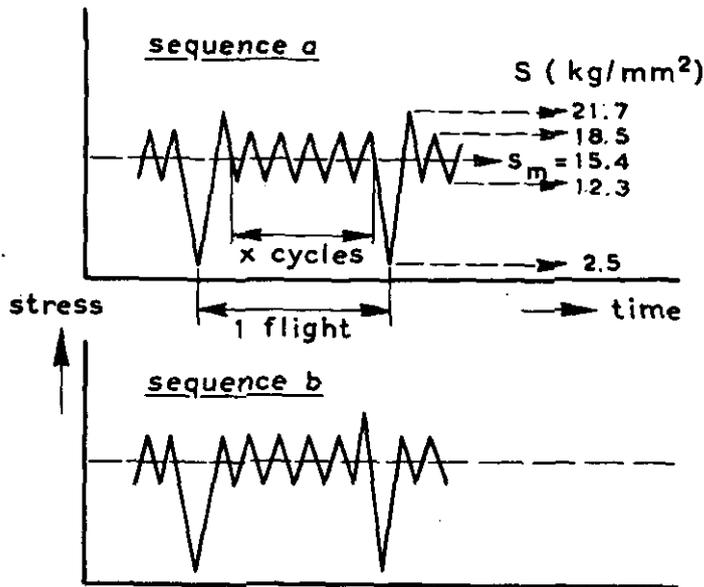
Spectrum	Material	S_1 (kg/mm^2)	N_1 at S_1 (kc)	N_3 at S_3 (kc)	$\frac{1}{\sum N}$ 2)	Number of tests
	7075	24.7	8	105	1.93	4
II	2024	18.6	62	410	1.90	4
	7075	19.7	20.6	250	1.11	5

1) S_1 is 50% and 40% of S_u of the lug for spectrum I and II respectively.

2) Arithmetic mean.

Fig. 27 Results of program-fatigue tests on lugs of two types of light alloys. Results reported by Wallgren and Svensson (ref. 56)

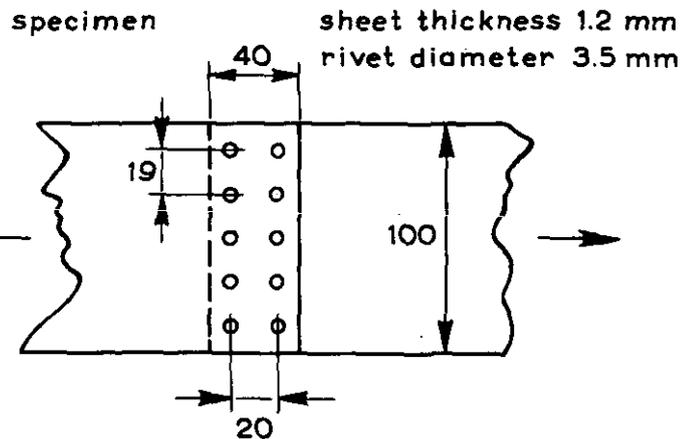
loadsequence:



Loadsequence represents gust loads and landing cycles. All gusts except one have the same amplitude. At the beginning of each flight a higher positive gust load was applied, its amplitude corresponding to an estimated frequency of once per flight. Tests were performed for different values of x (gust cycles per flight).

Loadsequence b is similar to sequence a, however, higher positive gust now at the end of the flight.

Riveted single lapjoint, with 2 rows of 5 rivets each.
 Sheet material: AU4G1 (2024)
 Endurances under constant-amplitude loading.
 Landing cycle: 2.5-21.7 kg/mm² → N=7360
 Gust cycle : 12.3-18.5 kg/mm² → N=205300



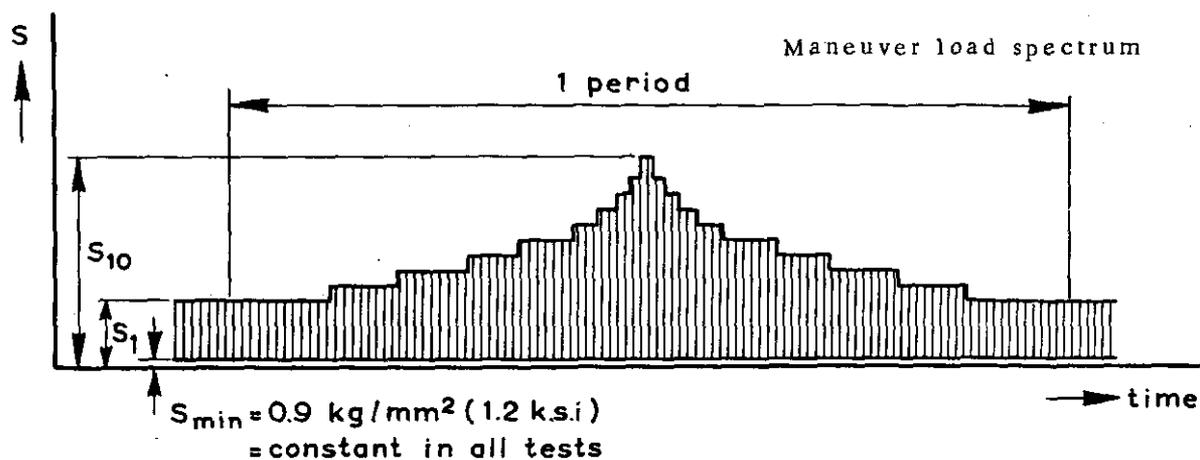
Test results :

Load sequence	Gust cycles per flight(x)	5	10	49	99	999
a	$\overline{\frac{n}{N}}$ 1)	0.53	0.51	0.44	0.37	0.63
	Total number of cycles	19740	30380	58400	59850	125800
	Number of flights	3290	2762	1168	598	126
b	$\overline{\frac{n}{N}}$ 1)	0.60	0.48	0.38	0.34	0.42
	Total number of cycles	22400	28350	51500	55431	84600
	Number of flights	3733	2577	1010	554	85

1) Each value is the mean of three tests.

Remark: For the damage calculations the landing cycle and the higher gust were considered as one load cycle for which $S_{max} = 21.7 \text{ kg/mm}^2$ and $S_{min} = 2.5 \text{ kg/mm}^2$ (stress in the sheet).

Fig. 28 Results of fatigue tests with landing-cycles on a riveted joint, reported by Barrois (ref.2).



Maximum of load cycles(g)	2.2	2.7	3.2	3.7	4.2	4.7	5.2	5.7	6.2	6.7
Number of cycles in 1 period	26	20	18	14	12	8	6	4	2	1

Three spectra have been applied differing only in the "stress per g".

Specimens



Material Extruded aluminum alloy DTD 363 A (AlZnCuMg)
For composition and properties see fig. 30
For the specimen : $S_u = 58.3 \text{ kg/mm}^2 (82.8 \text{ k.s.i.})$

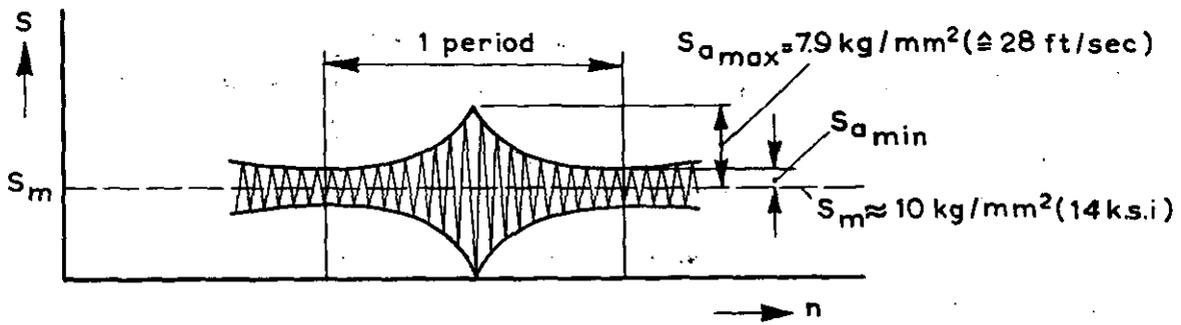
Test results

Program No.	Design factor	S_{10} (% S_u)	N at S_{10}	S_1 (% S_u)	N at S_1	Number of tests	$\frac{\sum n}{N}$ ²⁾
1	$S_u = 11g$	61.2	~ 100	20	14000	3	1.18
2	$S_u = 13g$	51.7	~ 300	16.9	50000	3	1.22
3	$S_u = 15g$	44.8	~ 500	14.7	160000	3	1.00
2a ¹⁾	$S_u = 13g$	56.7	~ 160	16.9	50000	4	1.00
2b ¹⁾	$S_u = 13g$	61.7	~ 100	16.9	50000	3	1.22
2c ¹⁾	$S_u = 13g$	66.7	~ 70	16.9	50000	3	1.19

1) The program was identical to program no. 2, only the highest load level has been increased to the value indicated in the third column.

2) Logarithmic mean.

Fig. 29 Program-fatigue tests on a AlZnCuMg extruded alloy, notched specimen, loaded by a maneuver spectrum. Tests reported by Fisher (ref.10).



The spectrum is an exponential gust frequency distribution, i.e. there is a linear relation between U (ft/sec) and the logarithm of the number of exceedings.

The slope may be defined as $\frac{\partial U}{\partial \log m} = \frac{15 \text{ ft/sec}}{-\log 100}$

Type of specimen

The specimen is axially loaded



Type of material

Extruded aluminum alloy DTD 363A

(Zn 4-8%, Cu ≤ 3%, Mg ≤ 4%, Mn ≤ 1%, Cr ≤ 1%)

Static properties

$S_{0.1} = 59.0 \text{ kg/mm}^2$ (84.0 k. s. i.)

$S_u = 62.8 \text{ kg/mm}^2$ (89.3 k. s. i.)

$\delta = 6.5 \%$

Program test results

In all tests $S_m = 10 \text{ kg/mm}^2$ and $S_{a_{max}} = 7.9 \text{ kg/mm}^2$ (28 ft/sec) with a corresponding endurance of ~ 4000 cycles.

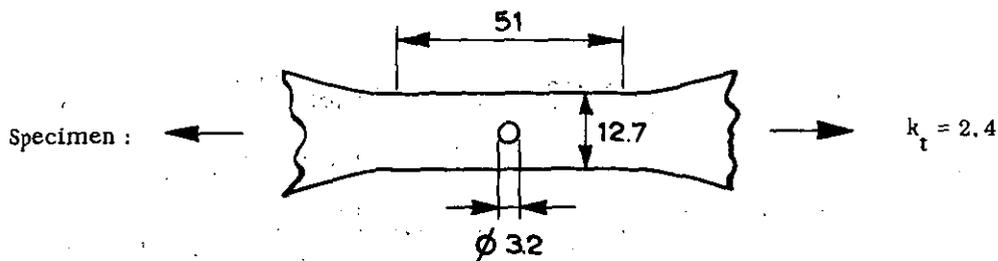
Program No.	Similarity with program no. 1	Number of tests	U_{min} (ft/sec)	$S_{a_{min}}$ (kg/mm ²)	N at $S_{a_{min}}$	Cycles in one period	Geometric mean number of periods	1) $\frac{\sum \frac{n}{N}}$
1	-	6	5	1.4	$> 10^7$	13000	10	1.18
2	period length halved	6	5	1.4	$> 10^7$	6290	19.3	1.05
3	lower S_a -values omitted	5	7.5	2.1	$\sim 500,000$	5530	13.7	1.47

1) Logarithmic mean.

Fig. 30 Program-fatigue tests on an AlZnCuMg extruded alloy, notched specimens, loaded by a gust spectrum.

Test reported by Fisher (ref. 11).

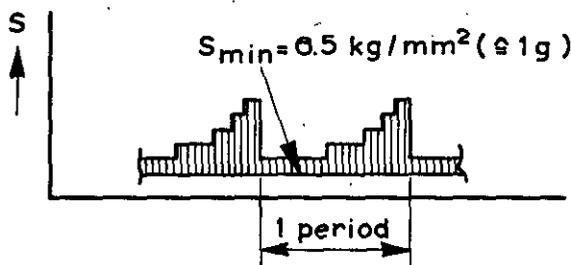
74



sheet thickness 1.6

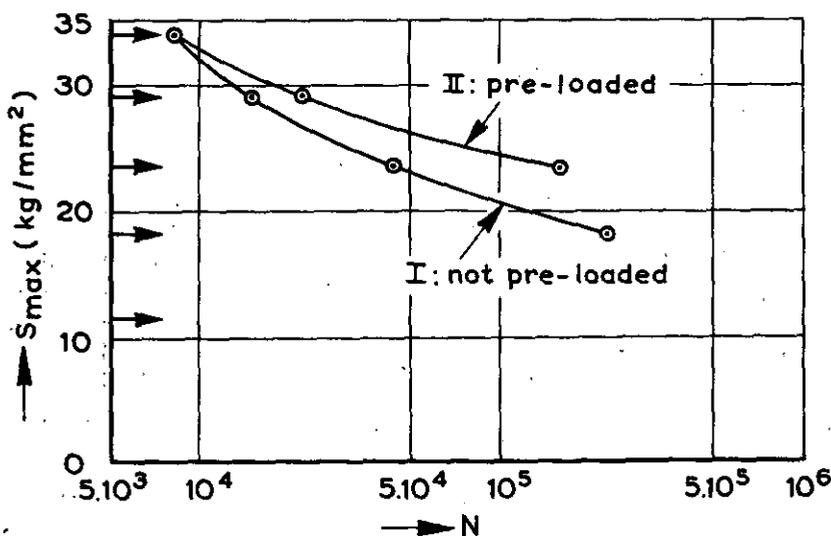
Material : 7075 T-6, bare sheet

Load sequence

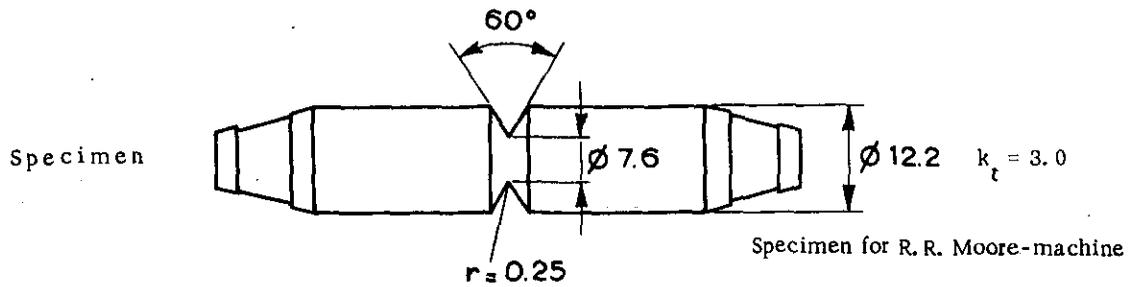


Stress level	Maximum load		$S_{max}^{1)}$ (kg/mm ²)	Number of cycles per period
	g	% limit load		
1	5.18	97.8	33.9	10
2	4.41	83.4	28.8	280
3	3.55	67.0	23.2	1100
4	2.73	51.5	17.9	3400
5	1.70	32.0	11.1	11100

1) stress on net section.



Arrows indicate the stress levels of the program tests.
For curve II each specimen was preloaded once at $S_{max} = 33.9 \text{ kg/mm}^2$, which is the highest stress level of the program tests.
Each point is the mean of 8 test results.



Material: 7075 extruded material

$S_u = 69 \text{ kg/mm}^2$, $S_{0.2} = 65 \text{ kg/mm}^2$, $\delta = 12\%$
 (Note very high S_u and $S_{0.2}$)

Type of loading: Continuously varying amplitude according to a gust spectrum.

Test results

Series	$S_{a \max}$ (kg/mm ²)	N at $S_{n \max}$ (kc)	$S_{a \min}$ (kg/mm ²)	N at $S_{a \min}$	$\frac{\sum n}{\sum N}$	Number of tests
1	23.9	11	1.04	∞	0.44	14
2	23.9	11	7.0	~2000000	0.46	15
3	20.4	19	6.3	∞	0.37	10

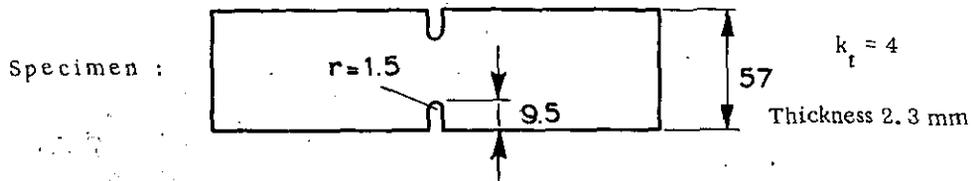
Fig. 32. Program fatigue tests on notched rotating beam specimens. Results reported by Hardrath, Utley and Guthrie (ref.26).

Results of program tests:

S-N-Data	$\frac{\sum n}{N}$ ¹⁾	$(\frac{n}{N})_{\min}$	$(\frac{n}{N})_{\max}$
Curve I	4.9		
Curve II	1.65	1.09	2.1

¹⁾ mean of 4 test results

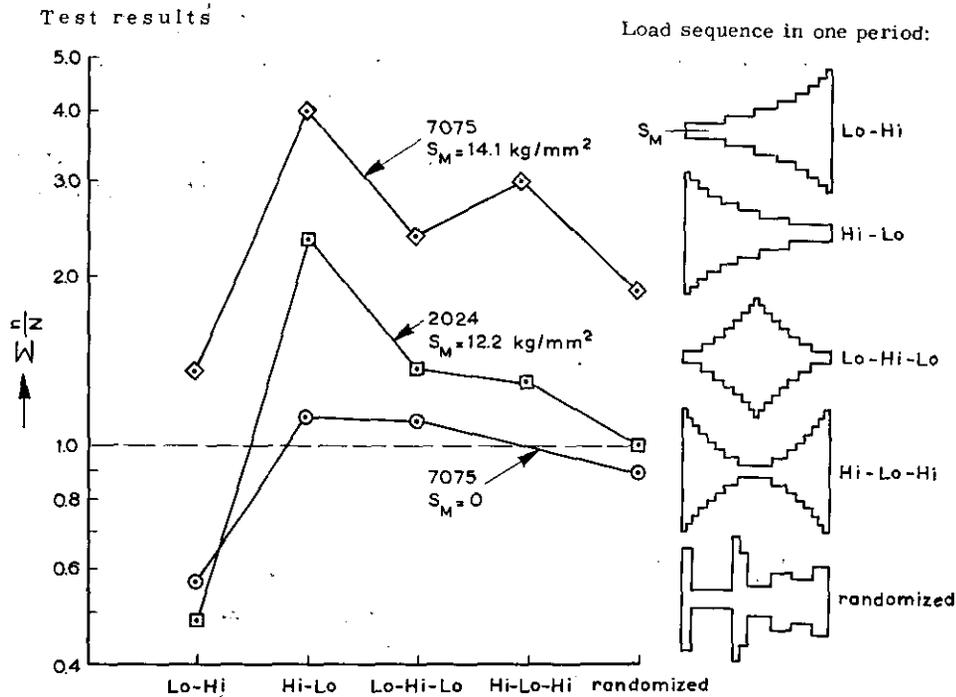
Fig. 31. Results of program-fatigue tests on notched sheet specimens. Results reported by Smith (ref.49).



Material : Bare 2024-T 3 sheet material. $S_u = 51 \text{ kg/mm}^2$, $S_{0.2} = 37 \text{ kg/mm}^2$, $\delta = 22 \%$.
 Bare 7075-T 6 sheet material. $S_u = 58 \text{ kg/mm}^2$, $S_{0.2} = 53 \text{ kg/mm}^2$, $\delta = 12 \%$.

Type of loading : Program-loading with 8 stress levels in accordance with a gust spectrum of Rhode and Donely (type A) (ref. 43). Constant S_m . For all tests the minimum S_a is below the fatigue limit. For the maximum value of S see the following table.

Material	S_m (kg/mm ²)	S_{max} (kg/mm ²)	N at S_{max} (cycles)
7075	14.1	33.4	480
7075	0	32.5	58
2024	12.2	29.1	750



Each point indicates the mean result of a group of specimens. Most groups consisted of 3 or 4 specimens. In the randomized sequence all cycles of one period with the same amplitude were applied in one batch. The order of succession of the 8 batches was selected at random.

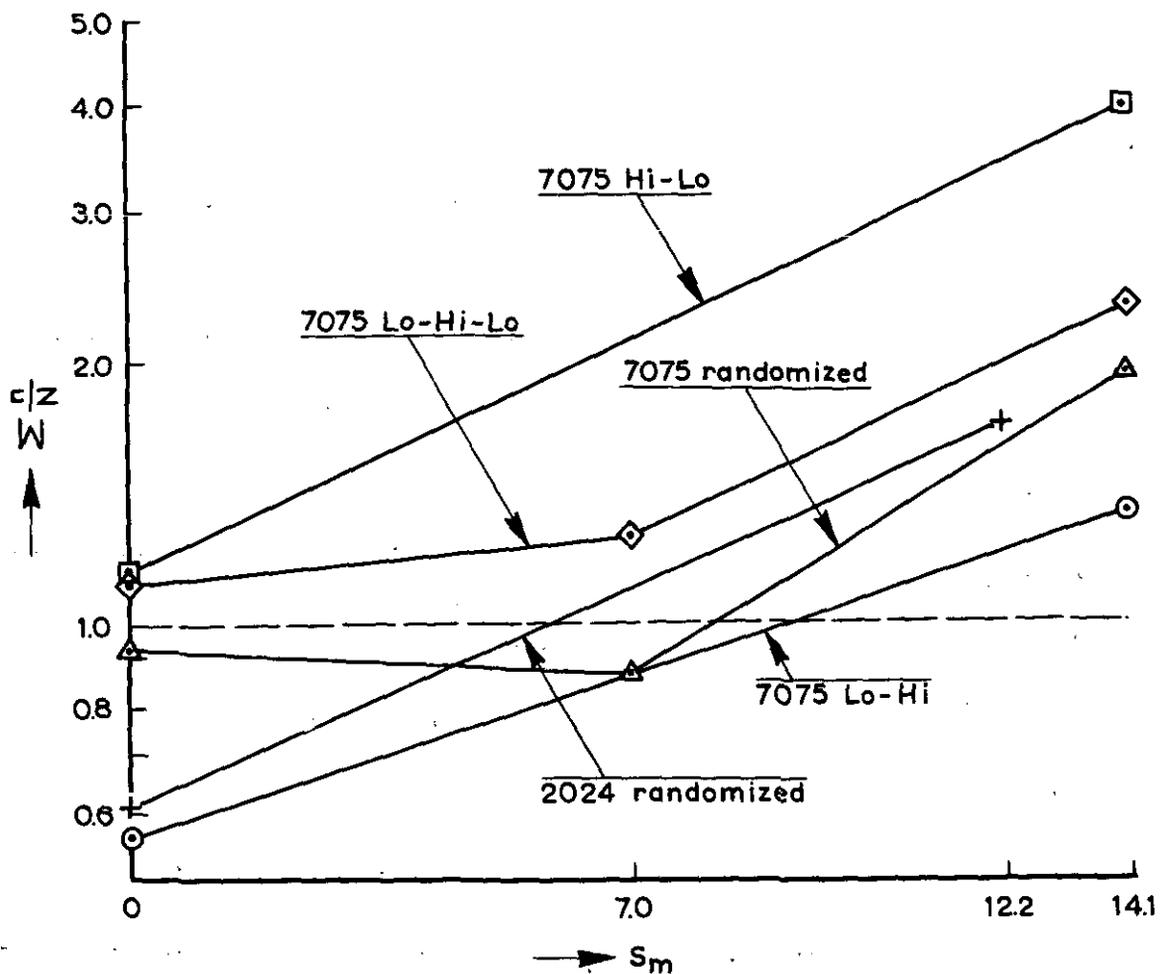
Fig. 33. The effect of the load sequence in program-fatigue tests on notched specimens. Results reported by Hardrath, Naumann and Guthrie (refs. 25 and 35)

Specimen : Side notches, $k_t = 4$.) More detailed
 Materials : 2024-T 3 and 7075-T 6) information in
 Type of loading : Gust spectrum, 8 stress levels, $S_m = \text{constant}$) fig. 33
 For all tests the minimum S_a is below the fatigue limit.

For the maximum value of S and the corresponding endurance see the following table.

Material	S_m (kg/mm ²)	S_{max} (kg/mm ²)	N at S_{max} (cycles)
7075	0	32.5	58
	7.0	33.1	140
	14.1	33.4	480
2024	0	28.4	58
	12.2	29.1	750

Test results :



Each point indicates the mean result of a group of 3 to 6 specimens. For each set of points the material and the type of load sequence are mentioned. The load sequences are shown in fig. 33.

Fig. 34. The effect of mean stress in program-fatigue tests on notched specimens.

Results reported by Hardrath, Naumann and Guthrie (refs. 25 and 35)

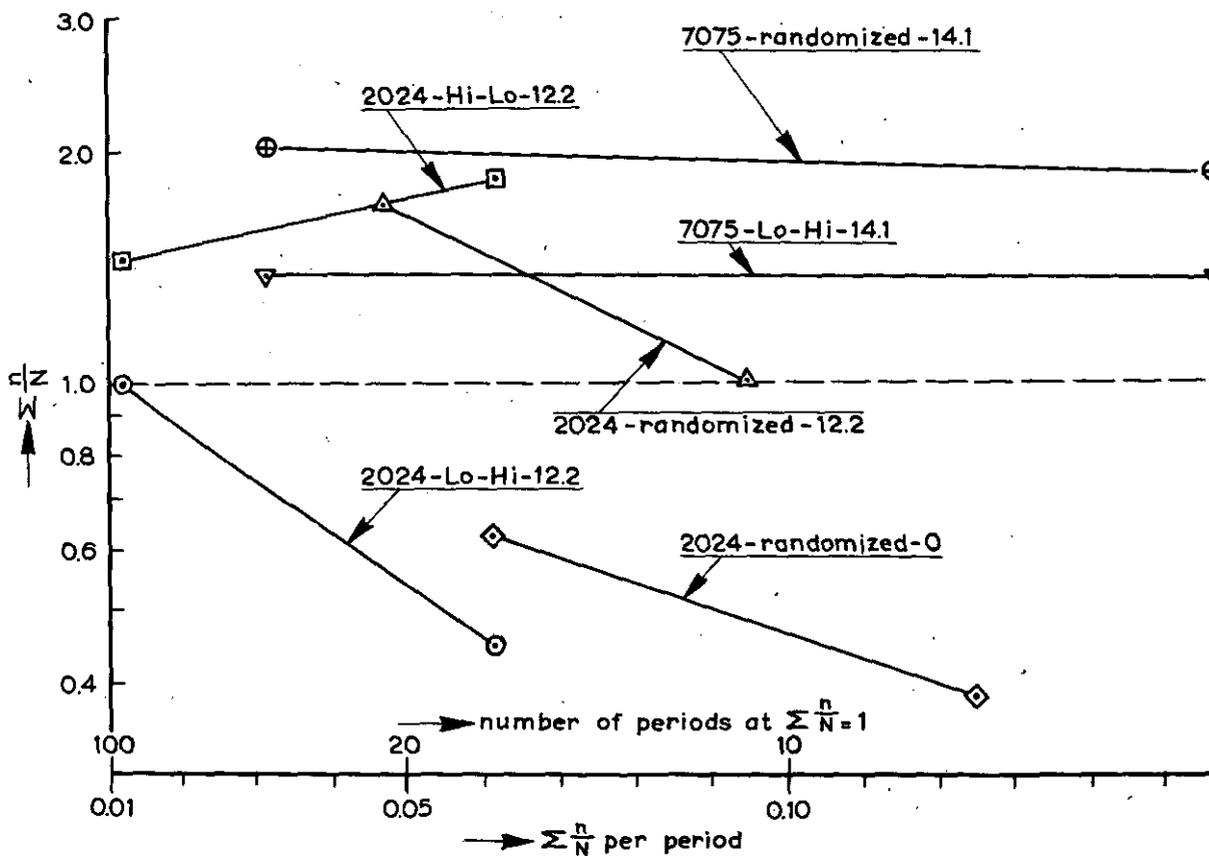
Specimen : Side notches, $k_t = 4$)
 Material : 2024-T3 and 7075-T6) More detailed
 Type of loading : Gust spectrum, 8 or 18 stress levels,) information
 $S_m = \text{constant}$) in fig. 33

For all tests the minimum S_a is below the fatigue limit

Testseries	S_m (kg/mm ²)	S_{max} (kg/mm ²)	N at S_{max} (cycles)	Number of stress levels	Gust spectrum type ¹⁾
7075-randomized - 14.1	14.1	33.4	480	8	A
7075-Lo-Hi					
2024-Hi-Lo	12.2	29.3	800	18	B
2024-Lo-Hi					
2024-randomized - 12.2	12.2	29.1	750	8	A
2024-randomized - 0	0	28.4	58	8	A

¹⁾ Type A and B according to Rhode and Donely (ref. 43).

Test results :



Each point indicates the mean result of a group of 2 to 10 specimens (average about 4 specimens).

Fig. 35. The effect of the period length on the endurance in program-fatigue tests on notched specimens. Results reported by Hardrath, Naumann und Guthrie (ref. 25 and 35).

Specimen : Side notches, $k_t = 4$)
 Materials : 2024-T3 and 7075-T6) More detailed
 Type of loading : Gust spectrum, 8 stress levels,) information in
 $S_m = \text{constant}$) fig. 33

The S_a -spectrum is derived from the gust spectrum by $S_a = (\frac{2}{3} S_u - S_m) \frac{V_i}{30}$

V_i is the gust velocity and a 30 fps gust is supposed to induce the design limit load. Note:
 (30 fps gust)_{U.S.} \approx (50 fps gust)_{I.C.A.O}

Further $S_m = \frac{2/3 S_u}{\text{design limit load factor}}$

S_u is the ultimate stress of the specimen. For the design limit load factor a value of about 2.5 has been used for both materials, leading to $S_m = 14.1 \text{ kg/mm}^2$ and 12.2 kg/mm^2 for 7075 and 2024 material resp. A comparison has also been made for $S_m = 0$, maintaining the same reduction method from gust spectrum to S_a -spectrum. Therefore the stress spectra for both materials are directly comparable from a design point of view. On an absolute basis they are different. For minimum and maximum values of S_a see fig. 34.

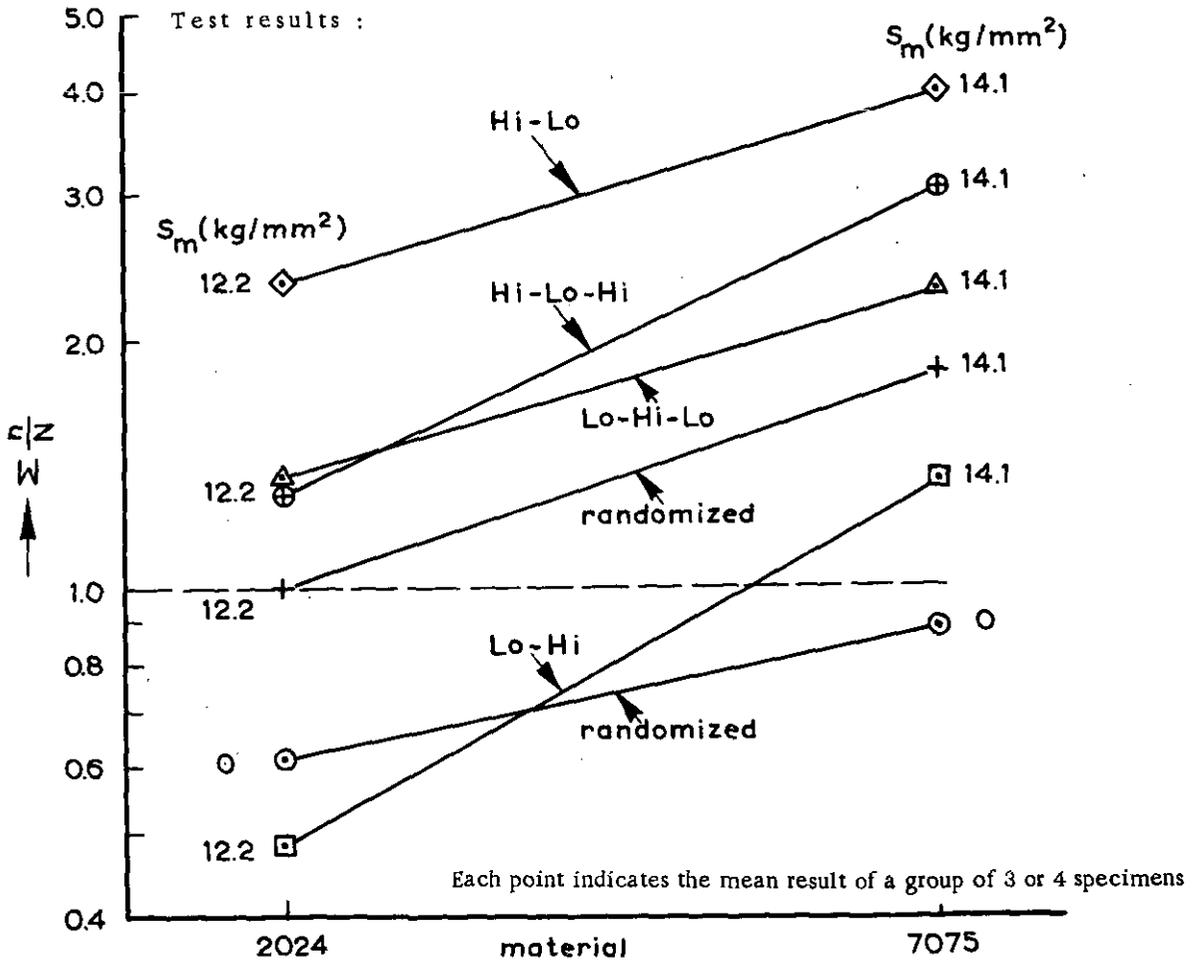
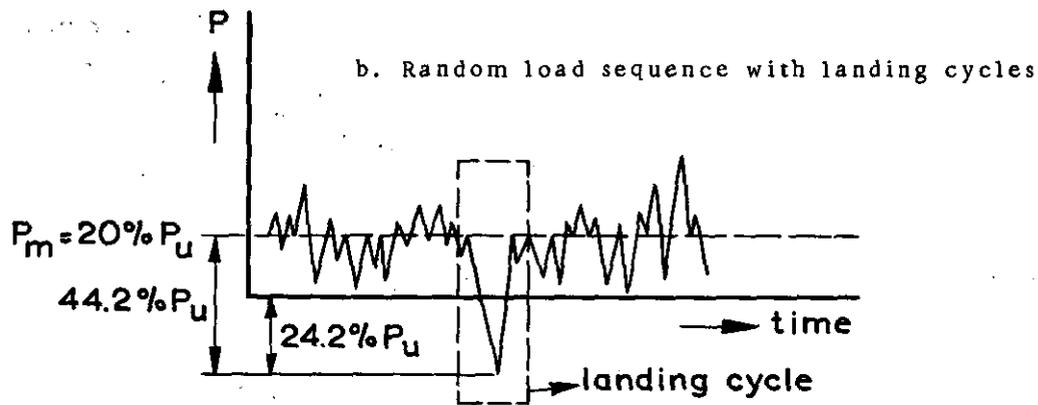
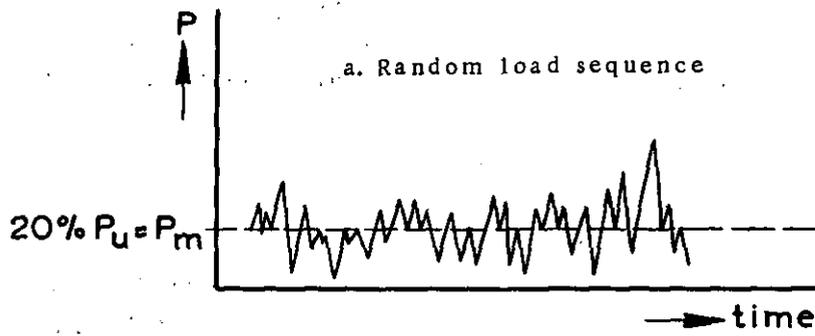


Fig. 36. Comparison of the results of program-fatigue tests on notched specimens of 2024 and 7075 material. Results reported by Hardrath, Naumann and Guthrie (ref. 25 and 35).



After each positive or negative gust the load returns to P_u . All gusts (half cycles) are selected at random out of the following gust spectrum (1 ft/sec gust $\hat{=} 1.44\% P_u$).

P_a (% P_u)	3.24	5.40	7.56	9.72	11.9	15.1	19.4	23.8	28.1	32.4	43
Number of occurrences in $5 \cdot 10^6$ cycles	2,753,000	1,182,000	643,100	260,800	94,520	51,100	11,390	2,207	513	107	41
Endurance	∞										$\sim 10^3$

The same gust spectrum is applied. However, after every 46 random load selections a landing cycle is added. This cycle consists of a downward load of $-24.2\% P_u$, which is preceded and followed by the smallest negative gust load.

Specimen : P 51 D Mustang-wing. Riveted 2024 structure with some cut outs. Cracks mainly originated in two areas the gun bay area and the tank bay area. Nominal stress level for both areas of the order of 28 kg/mm^2 with local peaks from 35 to 40 kg/mm^2 .

Test results		Calculations, irrespective of the location of the failure ¹⁾				Calculations taking into account the type and the location of the failure ²⁾				Number of half wings tested
Load sequence	Mean life (cycles) until final failure	$\sum \frac{n}{N}$ according to hypothesis H ³⁾ for final failure				$\sum \frac{n}{N}$, hypothesis H_1				
		H_1	H_2	H_3	H_4	Initial failure		Final failure		
						Gun bay	Tank bay	Gun bay	Tank bay	
a	3 275 000	1.23	0.38	-	1.07	0.84	2.02	2.66	1.34/	10
b	693 000	0.92	0.19	0.36	0.81	2.08	1.26	-	1.00	3 ⁴⁾

¹⁾ The endurances to final failure are used, irrespective of the location of this failure.

²⁾ The appropriate endurances were used which are related to the type of failure, i. e., initial failure or final failure, and to the location, i. e. the gun bay area or the tank bay area. Since only one type of final failure can occur in a specimen the endurance for the other type had to be estimated from the crack growth data. Therefore $\sum \frac{n}{N}$ - values in this category are not accurate.

³⁾ Hypothesis H_1 : damage calculations were based on the gust spectrum to which the landing cycles were added as negative gusts of $(20\% + 24.2\%) P_u$. The spectrum then becomes asymmetric. Positive and negative gusts with the same frequency of occurrence are combined to complete loadcycles, irrespective of their sequence in the tests (peak-method).

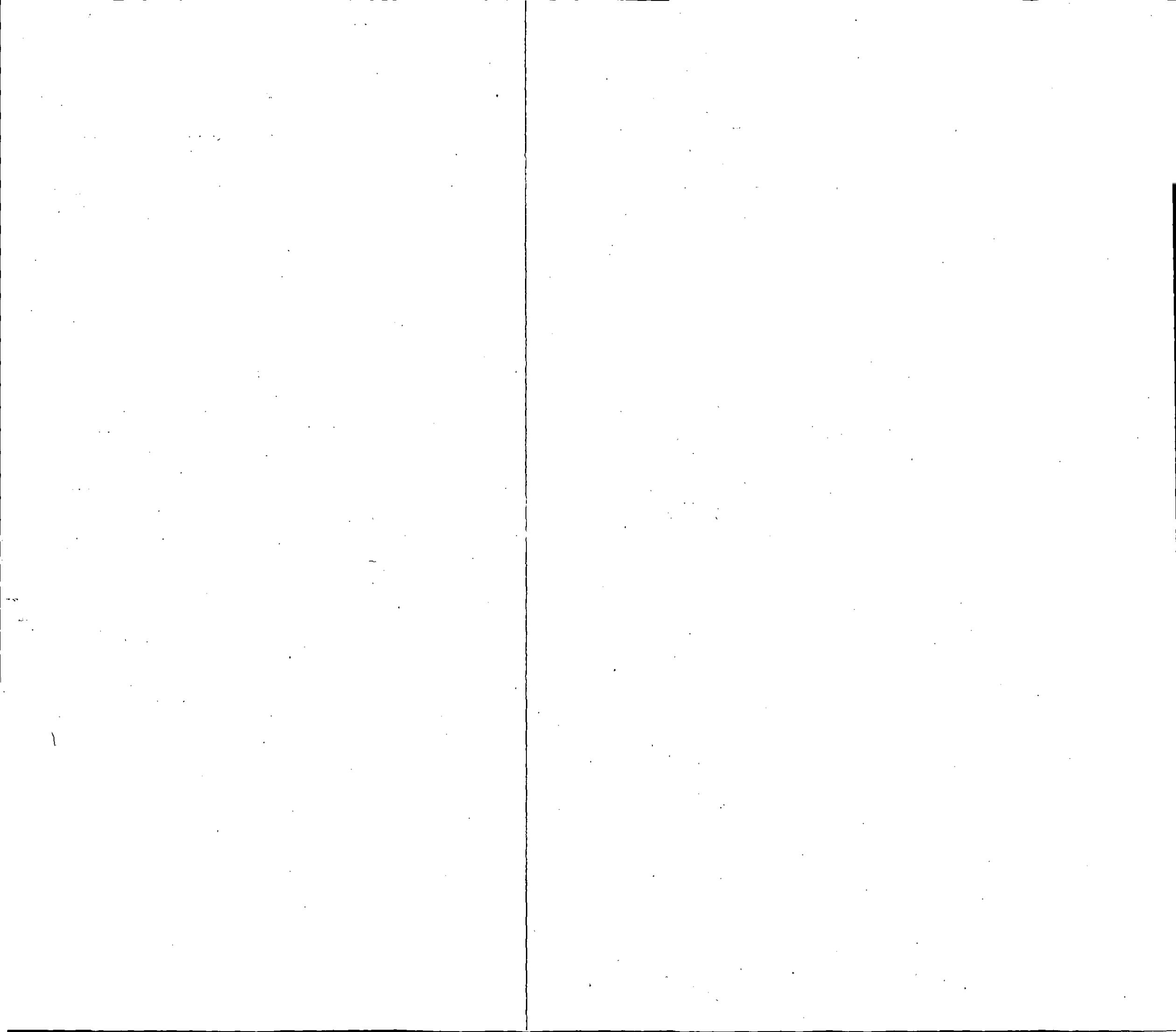
Hypothesis H_2 : damage calculations were based on the spectrum of load changes from each "peak" to the succeeding "trough" and vice versa (range-method).

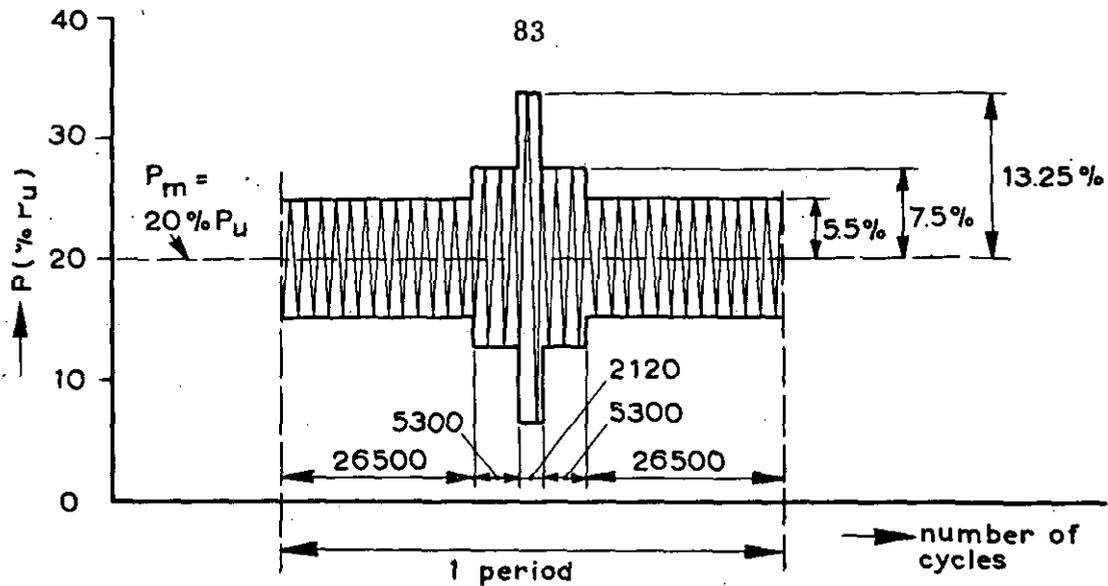
Hypothesis H_3 : similar to H_1 , however, landing loads were not introduced into the spectrum and their damage was calculated separately.

Hypothesis H_4 : similar to H_1 , however, N-values used are those for specimens preloaded by the maximum load occurring in the test ($63\% P_u$).

⁴⁾ 5 wings will be tested, the results of 3 half wings are available. In this testseries (b) the gun bay failure is largely suppressed as a final failure.

Fig. 37. Results of random load fatigue tests on Mustang wings, reported by Payne (ref. 37 and 38).





Load sequence

Three load levels were used. The second load level was selected at the estimated level of maximum damage. The number of load cycles per period at each level was about $\frac{1}{30}$ of the endurance until final failure at the relevant load level.

Specimen P 51 D Mustang-wing. Riveted 2024 structure with some cut outs. Cracks mainly originated in two areas, i. e. the gun bay area and the tank bay area.

Nominal stress level for both areas of the order of 28 kg/mm^2 with local peaks from 35 to 40 kg/mm^2 .

Test results:

Procedure of calculating		$\sum \frac{n}{N}$	$\overline{\sum \frac{n}{N}}$
Calculations irrespective of the location of the failure 1)			0.89
Calculations taking into account the type and the location of the failure 2)	Initial failure	Gun bay	2.62
		Tank bay	1.70
	Final failure	Gun bay	1.44
		Tank bay	1.03

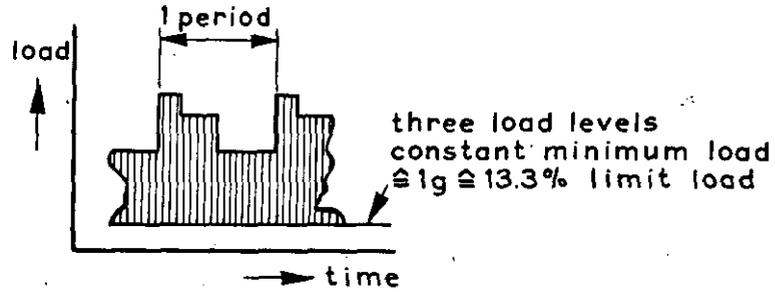
10 half wings were tested

- 1) The endurances to final failure were used, irrespective of the location of this failure.
- 2) The appropriate endurances were used which are related to the type of failure, i. e. initial failure or final failure and to the location, i. e. the gun bay area or the tank bay area. Since only one type of final failure can occur in a specimen the endurance for the other type had to be estimated from the crack growth data. Therefore $\sum \frac{n}{N}$ -values in this category are not accurate.

Fig. 38 Results of program-fatigue tests on Mustang wings, reported by Payne (ref. 38).

Specimen : Outer wing panel of a jet-fighter. 100% limit load $\hat{=}$ 7.5 g. Failure occurred in lower spar cap at tapped screw-holes, in the fatigue tests as well as in the static test. At this location the spar cap was slightly understrength.

Material : Lower spar cap material was 7075-T6.



2 fighter-maneuver spectra were applied

Stress level	Maximum load (% limit load)	N ¹⁾ (cycles) (final failure)	Number of cycles per period	
			Spectrum I	Spectrum II
1	100 %	1870	50	1
2	80 %	4000	75	1
3	60 %	16800	200	16

1) mean of 2 tests.

Test results

Two panels were tested with each of both spectra.

Spectrum	Total number of cycles	$\sum \frac{n}{N}$
I	7118	1.28
	8473	1.54
II	16413	1.61
	15164	1.49

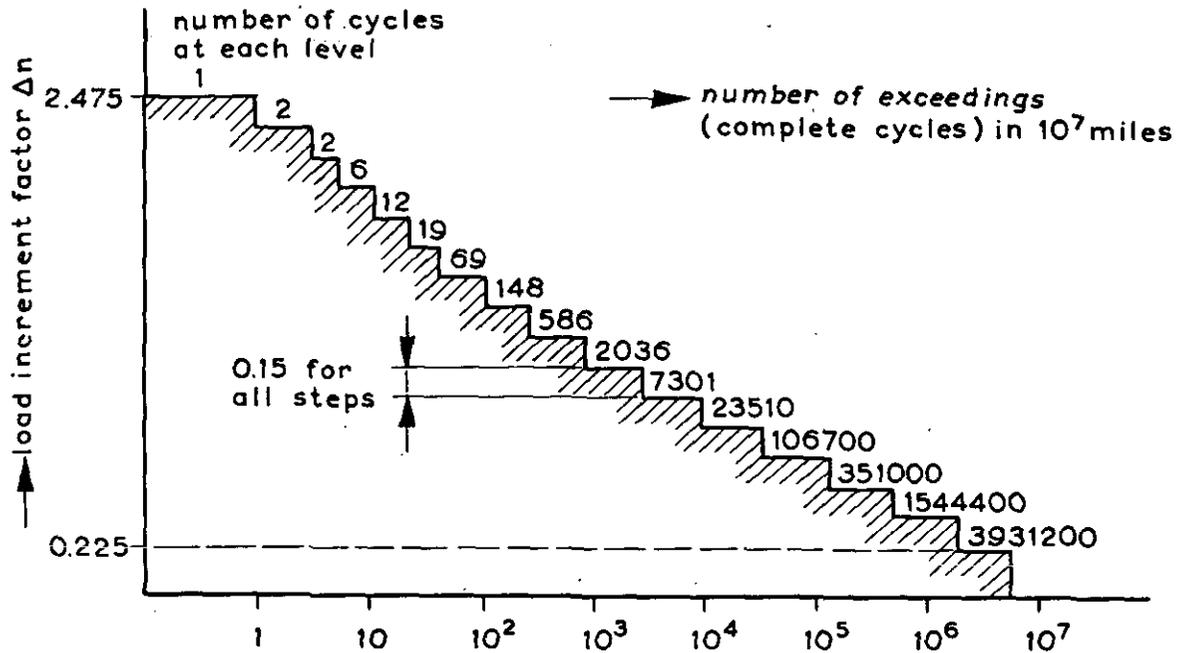
Fig. 39 Results of program-fatigue tests on outer wing panels.
Results reported by Carl and Wegeng (ref. 5).

Gust load spectrum approximated by 16 load levels.

The design ultimate load factor was 4.63 corresponding to a maximum stress in the tension surface of about 19 kg/mm^2 . (27 k. s. i.).

The mean load in the tests corresponds to 1.2 g instead of 1 g.

S_{max} at the highest level $\sim 15 \text{ kg/mm}^2$



- Specimen** : Complete wing of the C-46 airplane, consisting of the center section and two outer panels. The material is 2024. The distribution of the tension area over the skin, spar booms and stringers was 58.4%, 11.2% and 30.4% respectively. Most failures originated in the outer panels near the joint to the center section. In this area there were a number of cut outs.
- Test-program** : In one period of the program test about 60000 cycles ($= 10^5$ miles) were applied in accordance with the above loadspectrum. All load cycles of the same amplitude for one period were applied in one batch. The sequence of these batches was chosen at random for each period (randomized-step test). Load levels with a frequency lower than 1 in 60000 cycles (one in a period) were distributed at random over the program test.
- Test-results** : Three wings (6 outer panels) were tested under program loading. The locations and the number of cracks were found to agree better with the higher than with the lower constant-amplitude tests. Cracks leading to the final failure under program loading were only found in the higher constant-amplitude tests. Scatter in the time of first appearance of cracks was fairly large.

Based on the time to crack initiation regardless of the location the average result was

$$\overline{\frac{n}{N}} = 3.52$$

Based on the complete life the average value was $\overline{\frac{n}{N}} = 4.81$. This value may be somewhat flattered since in the lower constant-amplitude tests and the program tests different cracks have led to the final failure.

Fig. 40. Summary of randomized-step tests on commando wings. Results reported by Whaley (ref. 59).

Specimen: Wing of the C-46 airplane (Commando), see also fig. 40.

Loading in the laboratory: A load history was developed which corresponded as closely as possible to the flight operating experience of the C-46. Since calculations had indicated that loads due to such effects as landings, taxiing and maneuvers did not contribute significantly to the load history as compared to the contribution from rough air, the loading schedule was based on the most severe limits of the gust data of ref. 43, and the average value given in ref. 4 for the path ratio, i. e., ratio of flight distance in turbulence to total flight distance. The gust spectrum was converted to a randomized-step loading, see fig. 40.

Loading in service: It was understood from ref. 29 that no load records were obtained in service.

Results in the laboratory: From tests on 5 wings (10 outer wing panels) the first crack appeared on the average at 13000 hours of flight at 5 different locations.

Results in service: 4 aircraft with a flying time of 10000 to 14000 hours did not contain cracks.

4 other aircraft with a flying time of 19000 to 21000 hours had a total of 8 cracks at 4 different locations.

7 of these cracks at three different locations were indicated in the laboratory tests as being the first crack to appear. The 8th crack was found in the laboratory tests but not as the first crack to appear.

Fig. 41 The endurance of a wing structure in service as compared with laboratory test.

Results reported by Huston (ref. 29).

Specimen: In ref. 60 the joint is referred to as being a typical aircraft structural joint. The type of aircraft is not mentioned, probably it is a transport aircraft. The material is 7075-T6 Alclad for the skin in which the failure occurred near the ends of the top-hat stringers.

Loading in the laboratory: Six full-scale specimens were tested at $S_m = 9.1 \text{ kg/mm}^2$ (13.0 k. s. i.) and constant amplitudes to obtain the relevant S-N-curve from $N = 2 \cdot 10^3$ to $N = 10^6$.

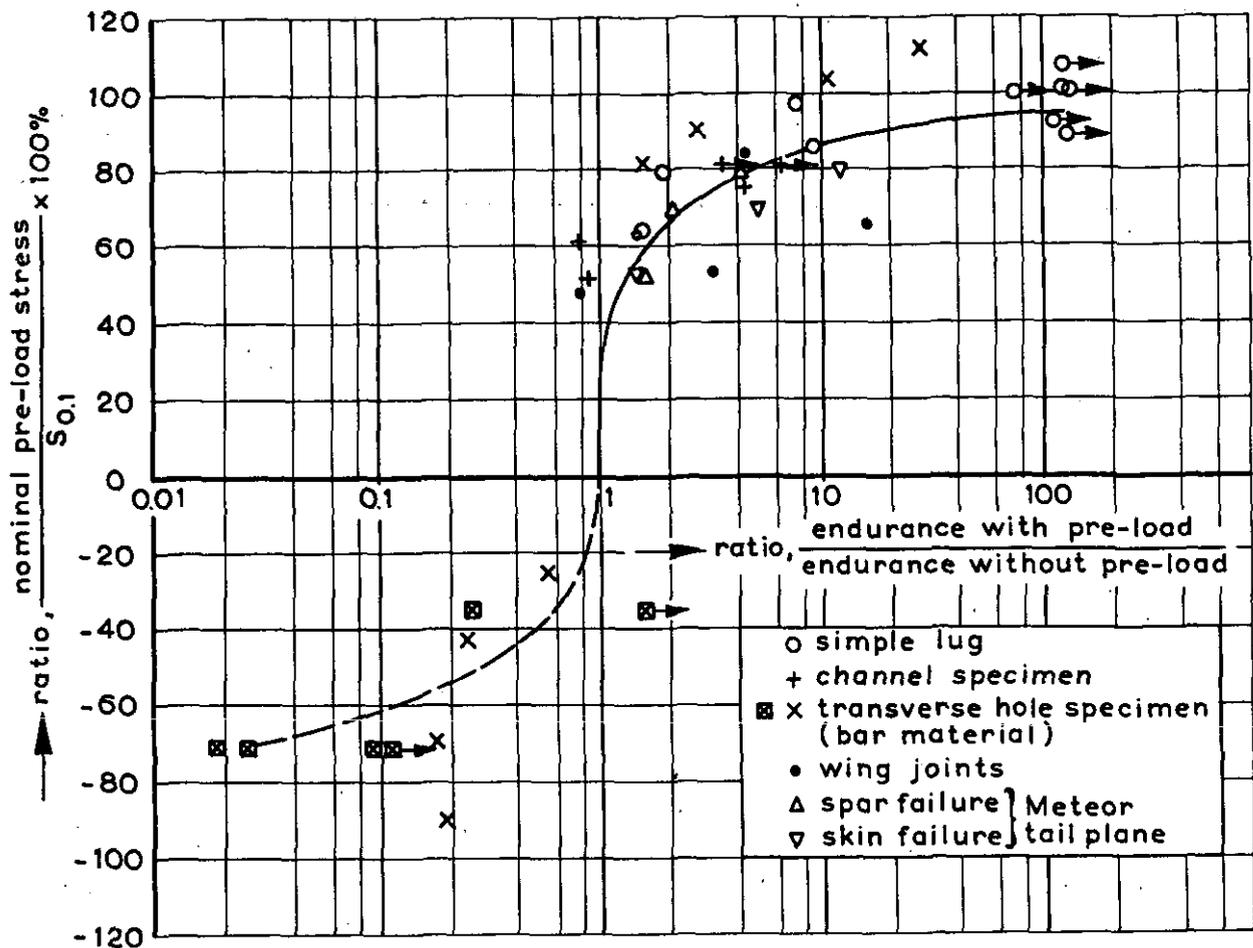
Loading in service: It was assumed that the gust loads were in accordance with the gust data of Rhode and Donely (NACA Wartime Report L 4 I 21, 1944, Curve A).

Calculated life: The calculation was based on the S-N-curve employing the linear cumulative damage rule and the above mentioned gust data ($1 \text{ ft/sec} = 0.19 \text{ kg/mm}^2 = 265 \text{ p. s. i.}$ which applies to typical flying conditions). Ground-to-air cycles and other types of loading were neglected. The result was 7900 flying hours.

Life in service: Cracking occurred between 10000 and 12000 hours.

Fig. 42. The endurance of a joint in service as compared with the life calculated from laboratory test results.

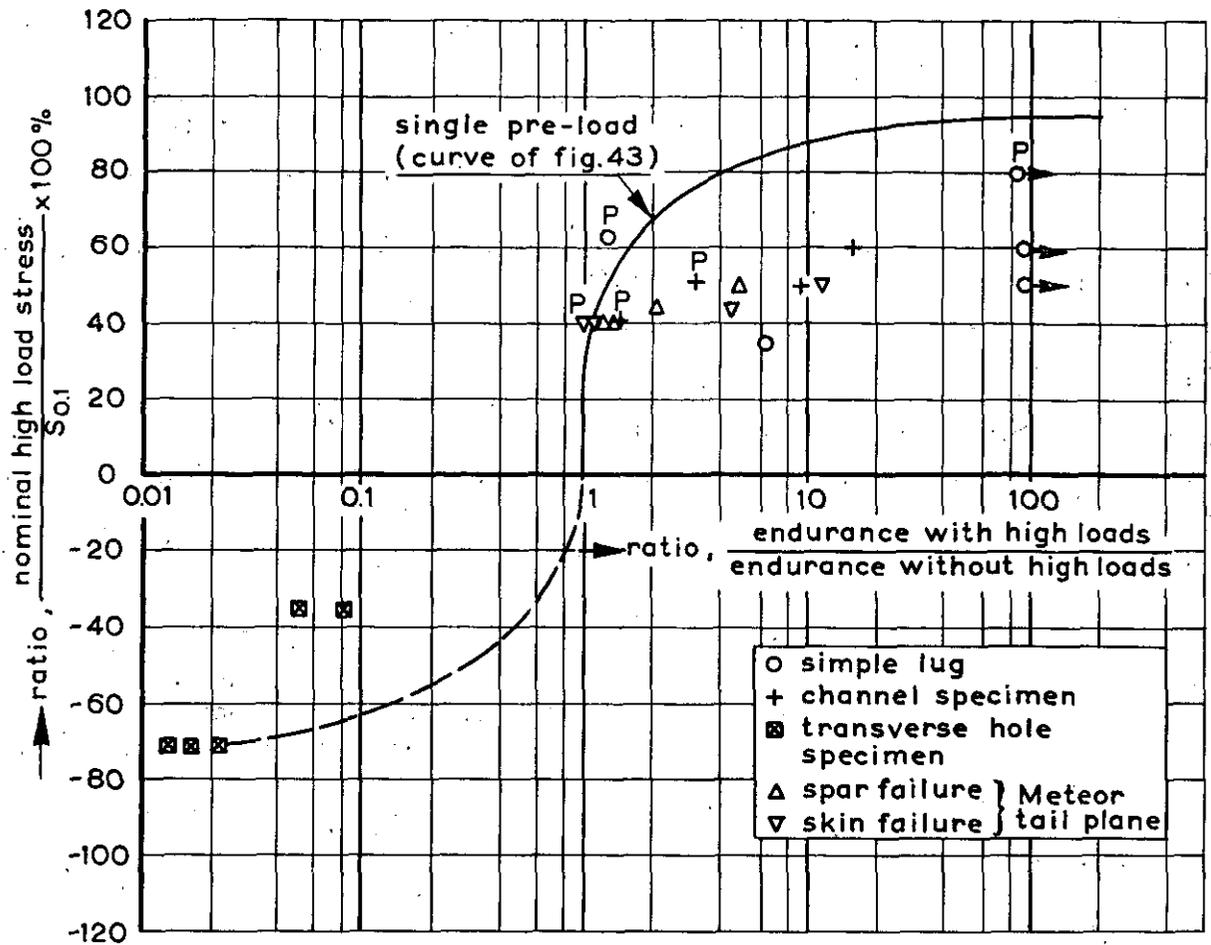
Results quoted in the Aircraft Fatigue Handbook (ref. 60).



All specimens were tested in fluctuating tension, except for Meteor tail-planes which were tested in bending at resonance. The mean stress was positive in all fatigue tests and ranged from 8.4-14.2 kg/mm². The stress amplitudes ranged from 2.7-5.4 kg/mm². The corresponding endurance for the not pre-loaded specimens varied from 79,000 to 850,000 except for six transverse hole specimens, indicated with \square for which N was 7,700,000.

$$1 \text{ kg/mm}^2 = 1.422 \text{ k. s. i.}$$

Fig. 43 The effect of one high pre-load on the endurance of different types of light alloy specimens.
Results reported by Heywood (ref. 28).



For some details on the test parameters see fig. 43.

Letter P near a point denotes that specimen had received ten high pre-loads.

All other points represent specimens subjected to periodic high loads, which were applied at intervals during the fatigue test, the load returning to the mean of the fatigue test after each application. The intervals of high loading were as follows:

At commencement of test, and every 20,000 cycles to 500,000 cycles

Then every 50,000 cycles to 1,000,000 cycles

Then every 100,000 cycles to 2,000,000 cycles

Then every 200,000 cycles to 4,000,000 cycles

Then test continued to failure without further overloads.

Fig. 44. Effect of 10 preloads and of periodic high loads on the endurance of different types of light alloy specimens.

Results reported by Heywood (ref. 28).

Specimen : Channel-section boom specimens machined from extruded material with rivets and bolt holes.

Material : D. T. D. 363 A (7075-type material).

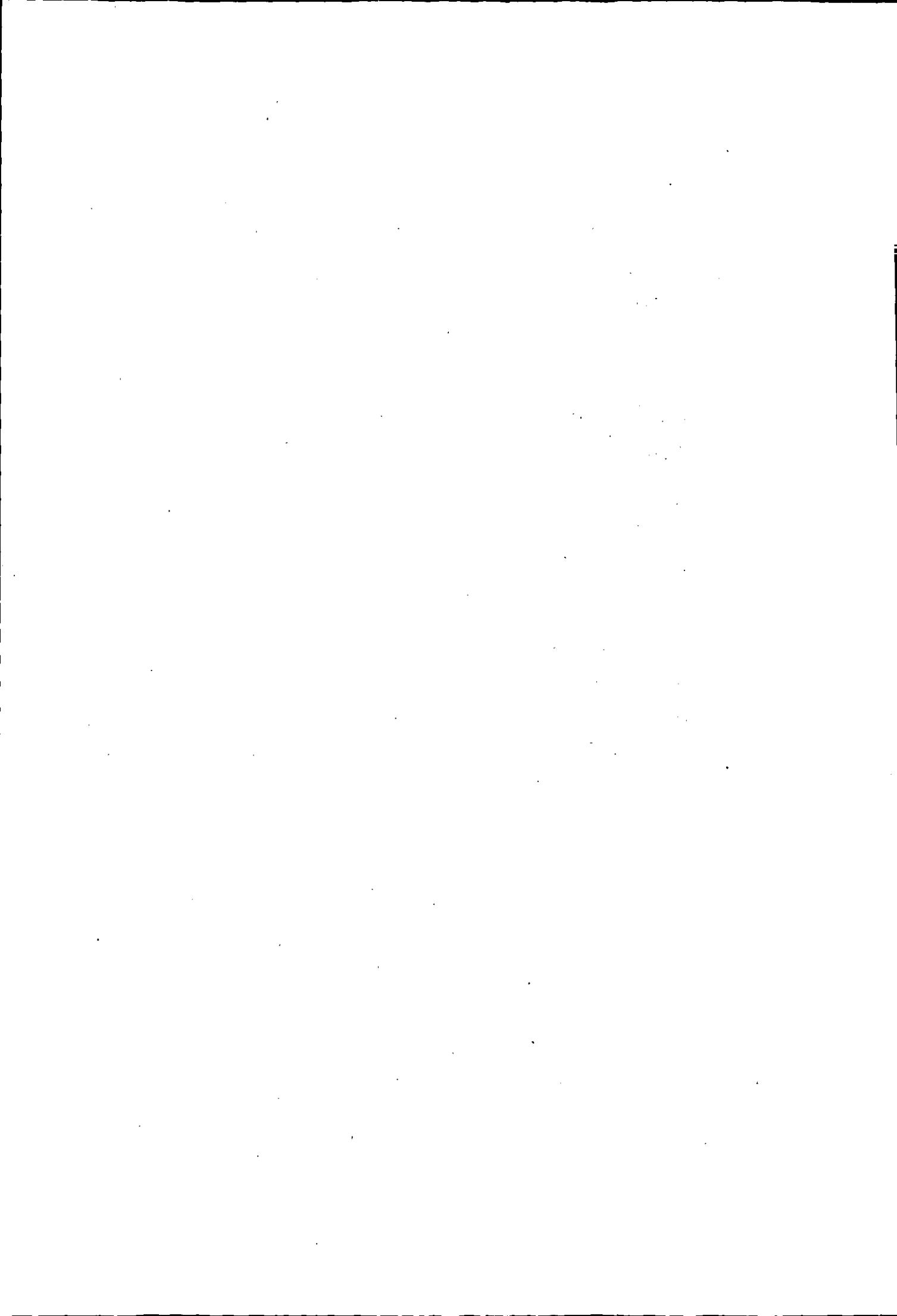
Type of loading : The effect of one preload and of periodic high loads on the endurance for a fatigue loading of $S_m + S_a = 11.2 + 4.3$ kg/mm² has been studied.

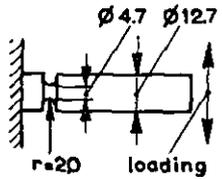
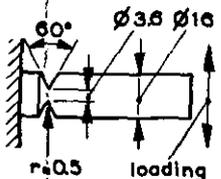
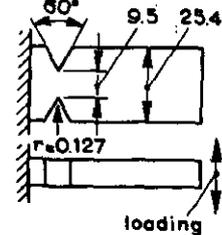
Test results :

Treatment	Magnitude of pre-load or high loads (kg/mm ²)	fatigue life (kc)	$\sum \frac{n}{N}$
No treatment	-	79 2)	1
One preload	26.4	69 3)	0.87
Periodic 1) high positive loads	26.0	769 3)	9.7
Periodic 1) high load cycles	$S_{max} = 26.0$ followed by $S_{min} = -5.2$	119 3)	1.50

- 1) For periodicity, see fig. 44.
- 2) Mean value of unspecified number of specimens.
- 3) Individual test results.

Fig. 45 The effect of high loads on the fatigue life of light alloy specimens according to results published by Heywood (ref. 28).



Investigator	Head and Hooke (ref. 26a)	Kowalewski (ref. 30)	Fralich (ref. 12)
Type of specimen	 circular cross section	 circular cross section	 plate specimen, t=0.3
Material	2024 extruded material	2024 extruded material	7075-T6 plate
Type of loading	plane bending ($S_m = 0$)	plane bending ($S_m = 0$)	plane bending ($S_m = 0$)
k_t	1.3	1.77	~ 4
Fatigue limit (kg/mm^2)	17.6	~ 13	6.0
σ_s Root mean square value ¹⁾ of stress induced by random noise (kg/mm^2)	7.9 and 8.6	Values in the range of 8 to 16	Values in the range of 2.8 to 13.4
Degree of irregularity ²⁾ N_0/N_1	0.84	0.91	~ 1
$\sum \frac{n}{N}$ - values	0.29 and 0.34	Values in the range of 0.7 to 1.1	Values in the range of 1.2 to 8. High values correspond to high values of σ_s

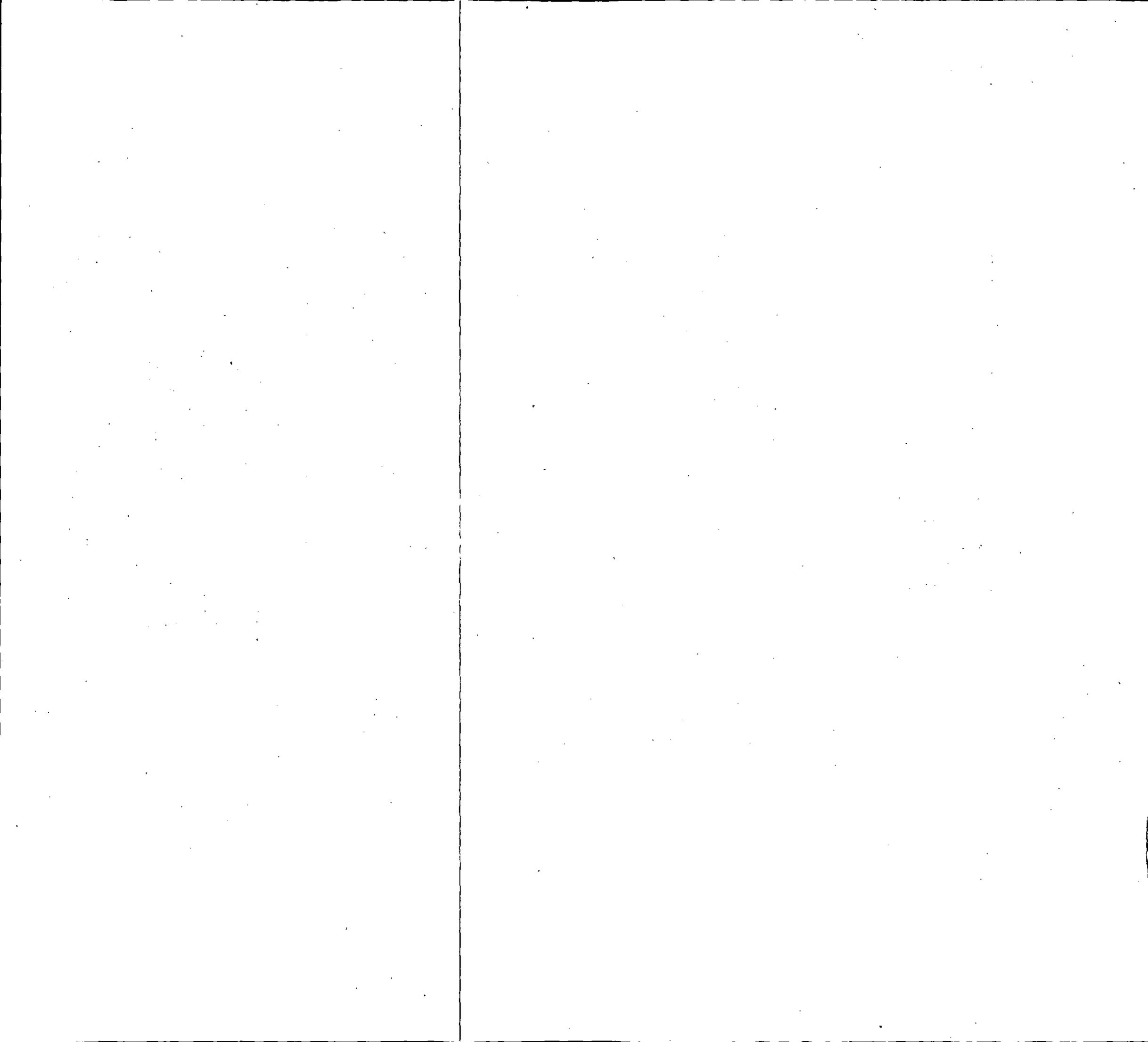
1) σ_s is the root-mean-square value of the stress amplitude as a function of time

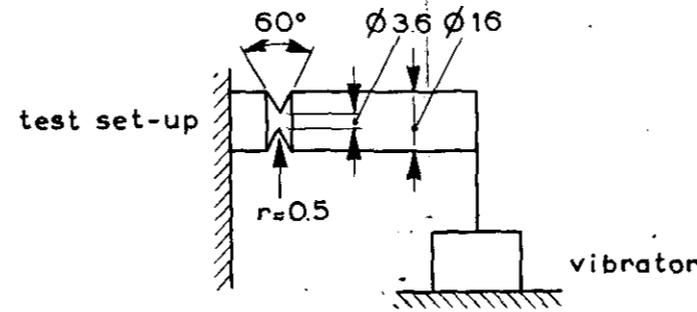
2) N_0 = number of times that $S = 0$ per second.

N_1 = number of times that S is a maximum or a minimum per second.

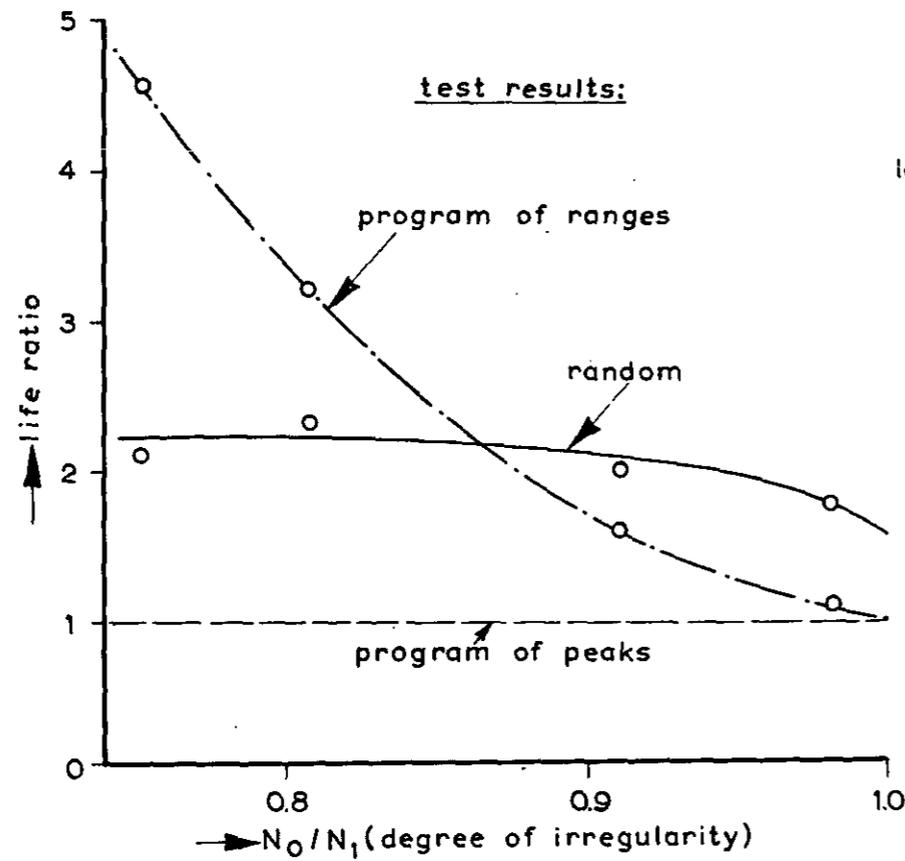
$$\sigma_s = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-\infty}^{+\infty} [S(t)]^2 dt$$

Fig. 46 Survey of random noise tests on small specimens.

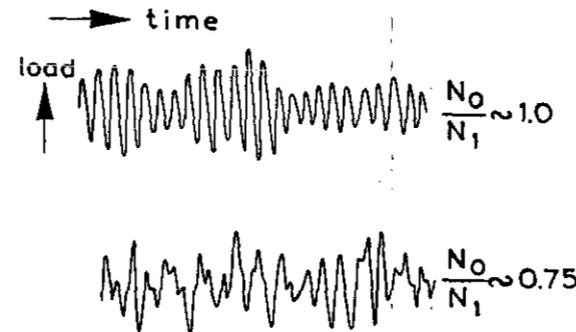




2024 specimen, $k_f = 1.77$. Specimen is loaded in plane bending ($S_m = 0$) by a vibrator. The input of the vibrator is either random noise or a programmed sequence with an increasing-decreasing order and 9 different stress levels.



N_0 = number of times that $S=0$ per second
 N_1 = number of times that S is a maximum or a minimum per second

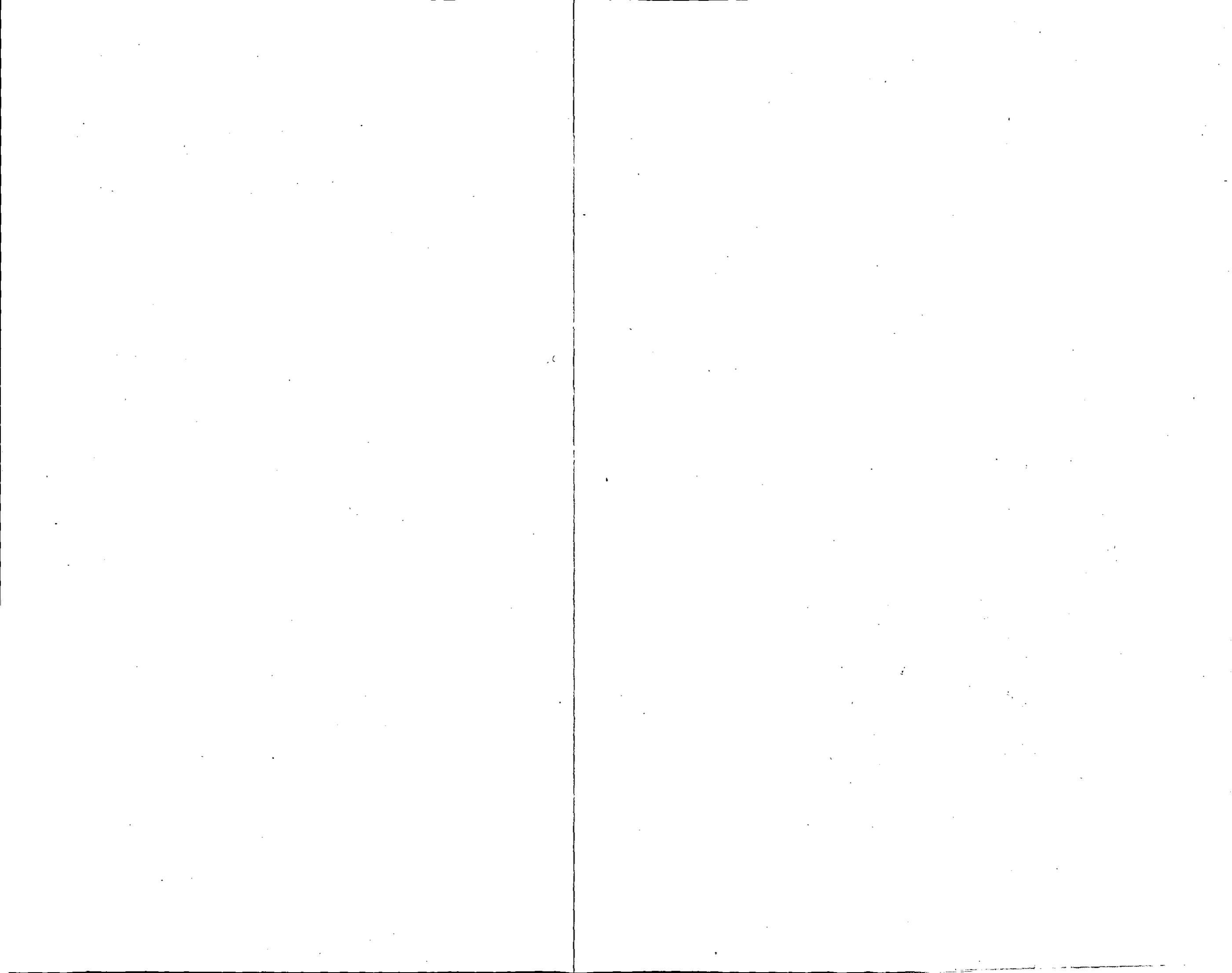


The irregularity is not completely determined by N_0/N_1 , it depends on the power spectrum.

- Type of loading:
- (1) Random noise with different degrees of irregularity (N_0/N_1). Root-mean-square value 10.6 kg/mm^2 .
 - (2) Program-fatigue loading, the program is derived from the random noise sequence with the peak-method.
 - (3) Program-fatigue loading, the program is derived from the random noise sequence with the range-method.

Test results : Vertical scale : The life under the program of peaks was chosen as equal to 1.
 Horizontal scale: $N_0/N_1 = 1$ low irregularity
 $N_0/N_1 \sim 0.7$ high irregularity

Fig. 47 Comparison of the fatigue life under random-noise loading and program-fatigue loading. Results reported by Kowalewski (ref. 30).

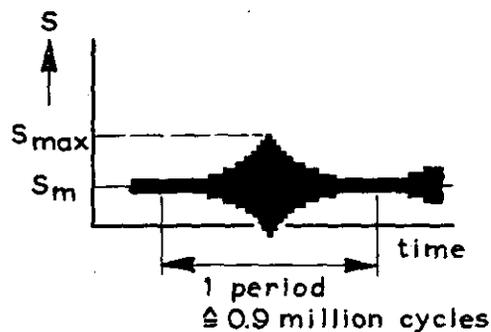


Specimen: Thin walled tube, notched by three holes, see fig. 19, specimen type b.

Material: Fliegwerkstoff 3115.5 (AlCuMg).

Type of loading: Specimen loaded in plane bending. Notches are in the outermost fibre of the tube, thus simulating axial loading

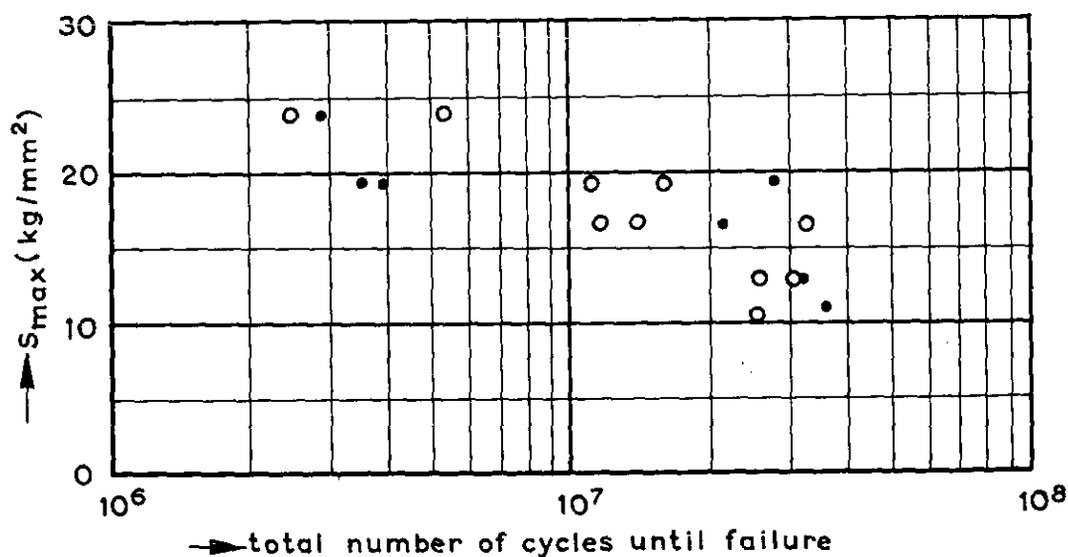
Loadsequence:



Program fatigue test with gust spectrum

$$S_m = \frac{S_{\max}}{3.4} \text{ in all tests}$$

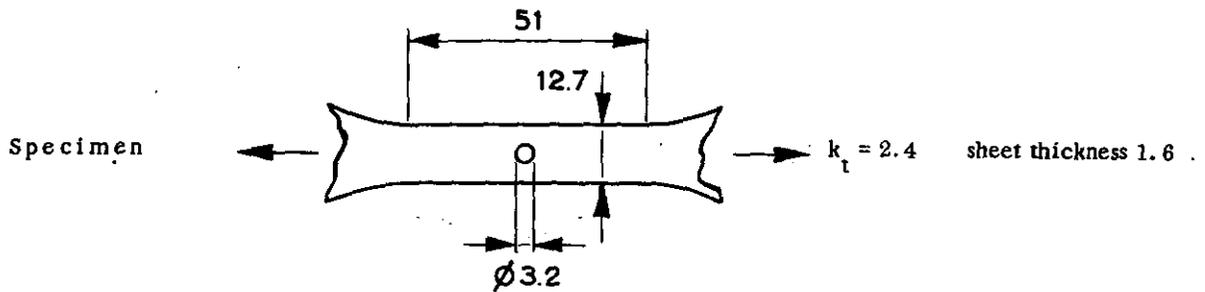
test results:



○ tests without rest periods

● tests with rest periods of 48 hours at intervals of 0.9 million cycles (= 1 period).

Fig. 48 The effect of rest periods on the endurance in program fatigue tests. Results reported by Gassner (ref. 37).



Material : 7075 T-6 bare sheet

Type of loading : Constant-amplitude testing at $S_m \pm S_a = 14.1 \pm 9.1 \text{ kg/mm}^2$.

Tests with and without preload. Preload stress = 34 kg/mm^2 .

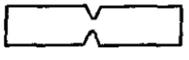
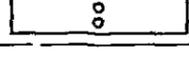
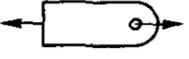
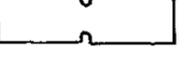
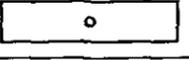
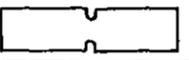
Test results :

Preload	Moment of fatigue tests	\bar{N} ²⁾ (kc)
no	immediately	92
yes	immediately	241
no	half a year later	94
yes	half a year later ¹⁾	220

1) i.e., half a year after preloading.

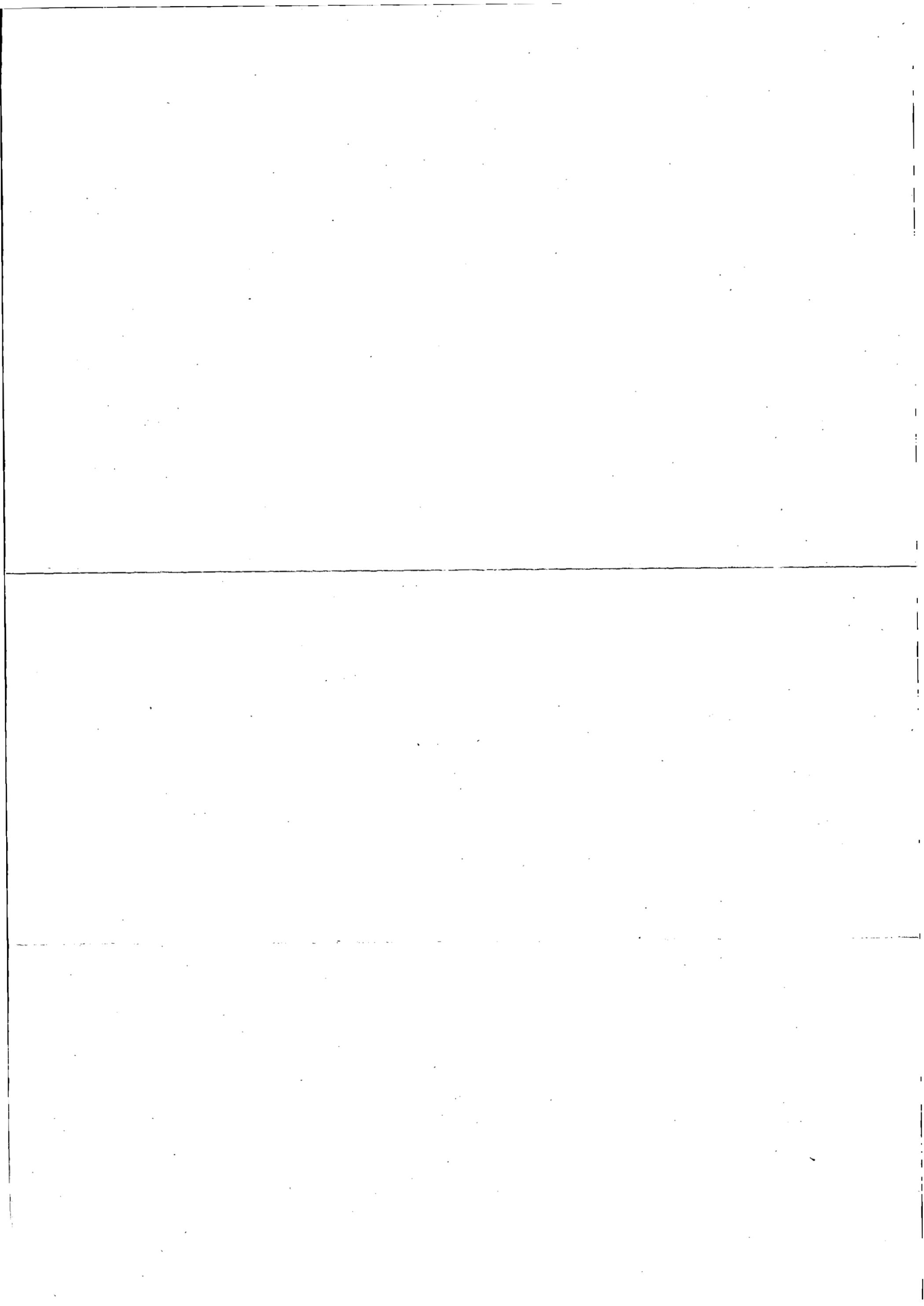
2) average of at least 4 specimens.

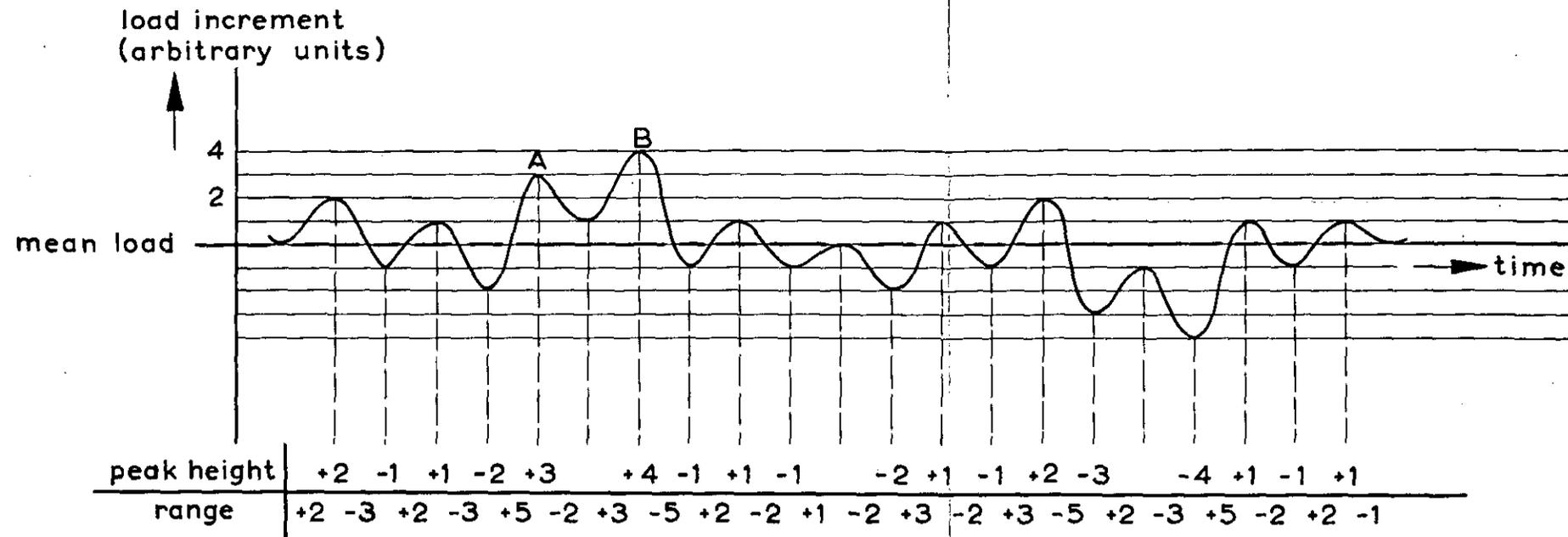
Fig. 49. The influence of a long rest period on the beneficial effect of a preload. Results reported by Smith (ref.49).

Type of specimen	Type of material	Type of loading	Type of load spectrum	Stress	Range of ¹⁾ $\sum \frac{n}{N}$ -values	Investigator	More details given in figure
	2024	rotating beam	Gust spectrum Maneuver spectrum	$S_m = 0$	0.7 - 0.9 1.0 - 1.2	Gassner	23
	7075	rotating beam	Gust spectrum	$S_m = 0$	~ 0.4	Hardrath et al.	32
	2024	plane bending	random noise	$S_m = 0$	0.3	Head and Hooke	49
	2024	plane bending	random noise	$S_m = 0$	0.7 - 1.1	Kowalewski	49
	7075	plane bending	random noise	$S_m = 0$	1.2 - 8	Fralich	49
	2024	plane bending	Gust spectrum Gust and Maneuver spectrum	$S_m > 0$	0.5 - 1.8 0.6 - 0.9	Gassner	19 20
Tube with 3 holes	2024	axial	Gust spectrum Gust and Maneuver spectrum	$S_m > 0$	0.3 - 2.0 0.7 - 1.8		19 20
	2024	axial	Gust spectrum	$S_m > 0$	0.9 - 3.0	Wällgren	24
Riveted joint	2024	axial	Gust spectrum Maneuver spectrum	$S_m > 0$ $S_{min} = \text{constant} > 0$	0.6 - 2.3 1.2 - 2.3		
Riveted joint	7075	axial	Gust spectrum Maneuver spectrum	$S_m > 0$ $S_{min} = \text{constant} > 0$	1.3 - 3.6 1.4 - 2.5		
Bolted joint	2024	axial	Maneuver spectrum	$S_{min} = \text{constant} < 0$	3.5	Wällgren and Petrelius	25
	7075	axial		$S_{min} = 0$	4.1	Wällgren	26
	2024 7075	axial	Maneuver spectrum	$S_{min} = 0$	1.3 - 1.9 1.1 - 1.9	Wällgren and Svensson	27
Riveted joint	2024	axial	Constant S and ground-to-air cycle	$S_m > 0$	~ 0.5	Barrois	28
	7075	axial	Maneuver spectrum Gust spectrum	$S_{min} \sim 0$ $S_m > 0$	1.0 - 1.2 1.0 - 1.5	Fisher	29 30
	7075	axial	Maneuver spectrum	$S_{min} = \text{constant} > 0$	4.9	Smith	31
Riveted joint	7075 2024	axial	Gust spectrum	$S_m > 0$	0.6 - 1.1 1.3 - 2.9	Schijve and Jacobs	14 to 16
	7075 2024	axial	Gust spectrum	$S_m = 0$ $S_m > 0$ $S_m = 0$ $S_m > 0$	0.6 - 1.1 0.9 - 4.0 0.4 - 0.6 0.5 - 2.4	Naumann, Hardrath and Guthrie	33 to 36
Mustang wing	2024	Wing bending	Random gust spectrum Random gust spectrum and ground-to-air cycles 3-level test	$S_m > 0$	0.8 - 2.7 1.0 - 2.1 1.0 - 2.6	Payne et al.	37 37 38
Wing structure	7075	Wing bending	Maneuver spectrum	$S_{min} = \text{constant} > 0$	1.3 - 1.6	Carl and Wegeng	39
Commando wing	2024	Wing Bending	randomized gust spectrum	$S_m > 0$	3.5	Whaley	40

¹⁾ The minimum and the maximum values of averages of test series are presented.

Fig. 50 Survey of investigations on program-fatigue testing of notched light alloy specimen .





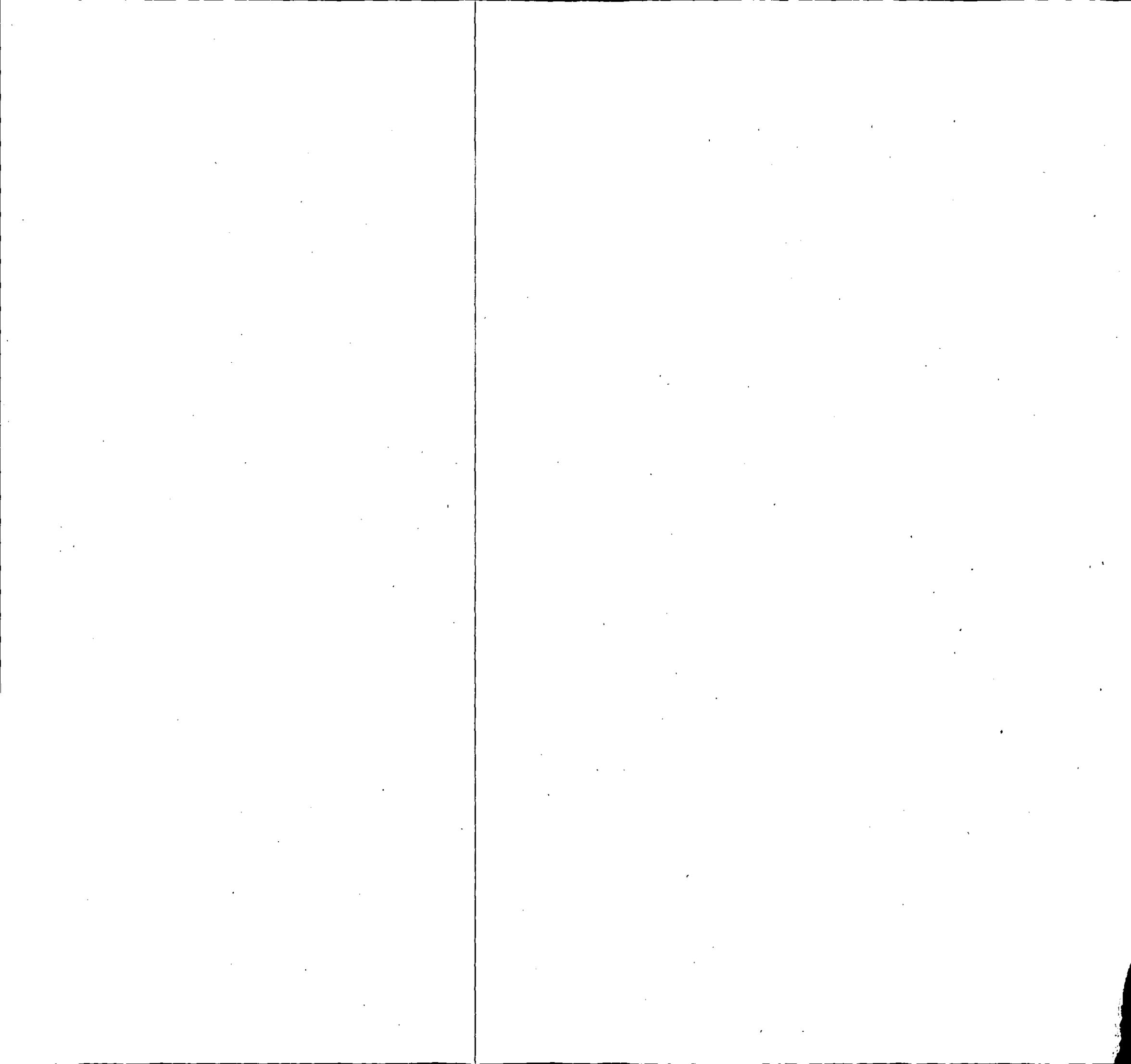
Peak-method : Peaks of equal height and opposite sign are combined to form complete cycles with amplitude equal to the peak height.
 Range method : Ranges of equal size and opposite sign are combined to form complete cycles with amplitude equal to half the size of the range. Effect of mean load is disregarded.

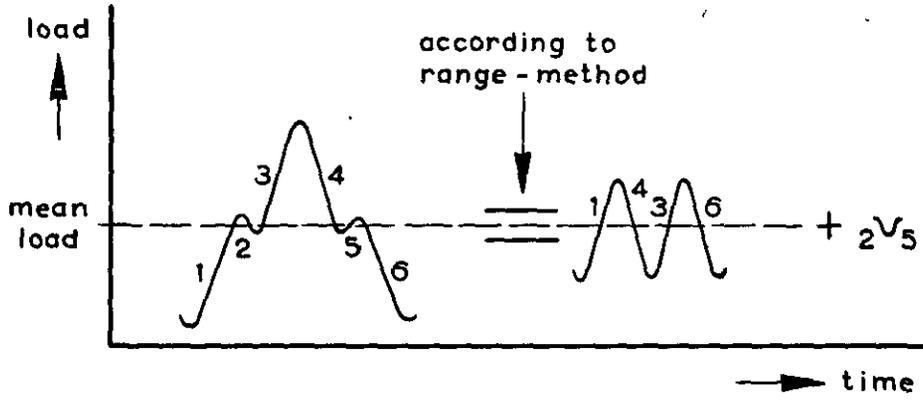
Peak height or load range	-5	-4	-3	-2	-1	+1	+2	+3	+4	+5
Number of peaks	-	1	1	2	5	5	2	1	1	-
Number of ranges	2	-	3	5	1	1	5	3	-	2

Results of the reduction

Load amplitude	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	4	
Number of cycles	Peak-method	-	5	-	2	-	1	1
	Range-method	1	5	3	-	2	-	-

Fig. 51. Reduction of a load-time history to load cycles according to the peak-method and the range-method





Note : If the small variations are neglected the decomposition implies doubling the number of cycles and halving its amplitude.

Fig. 52. Result of the decomposition of load variations in an extreme case according to the range-method.

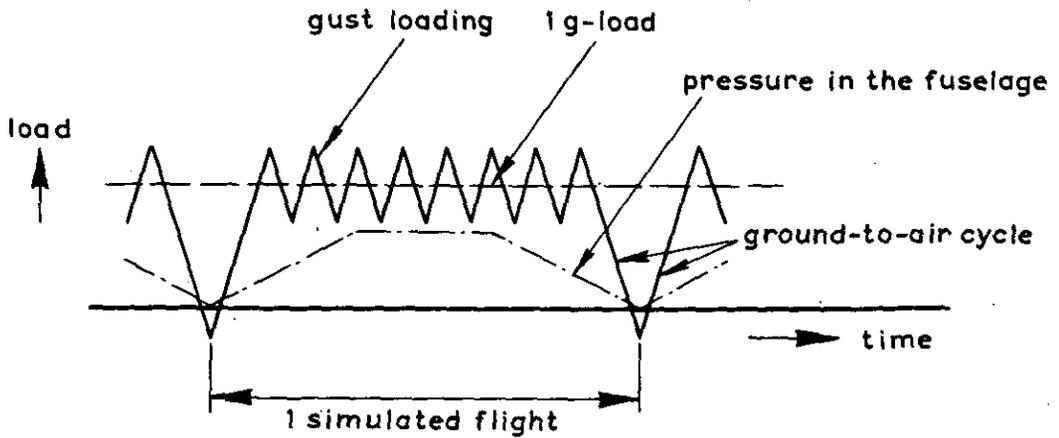
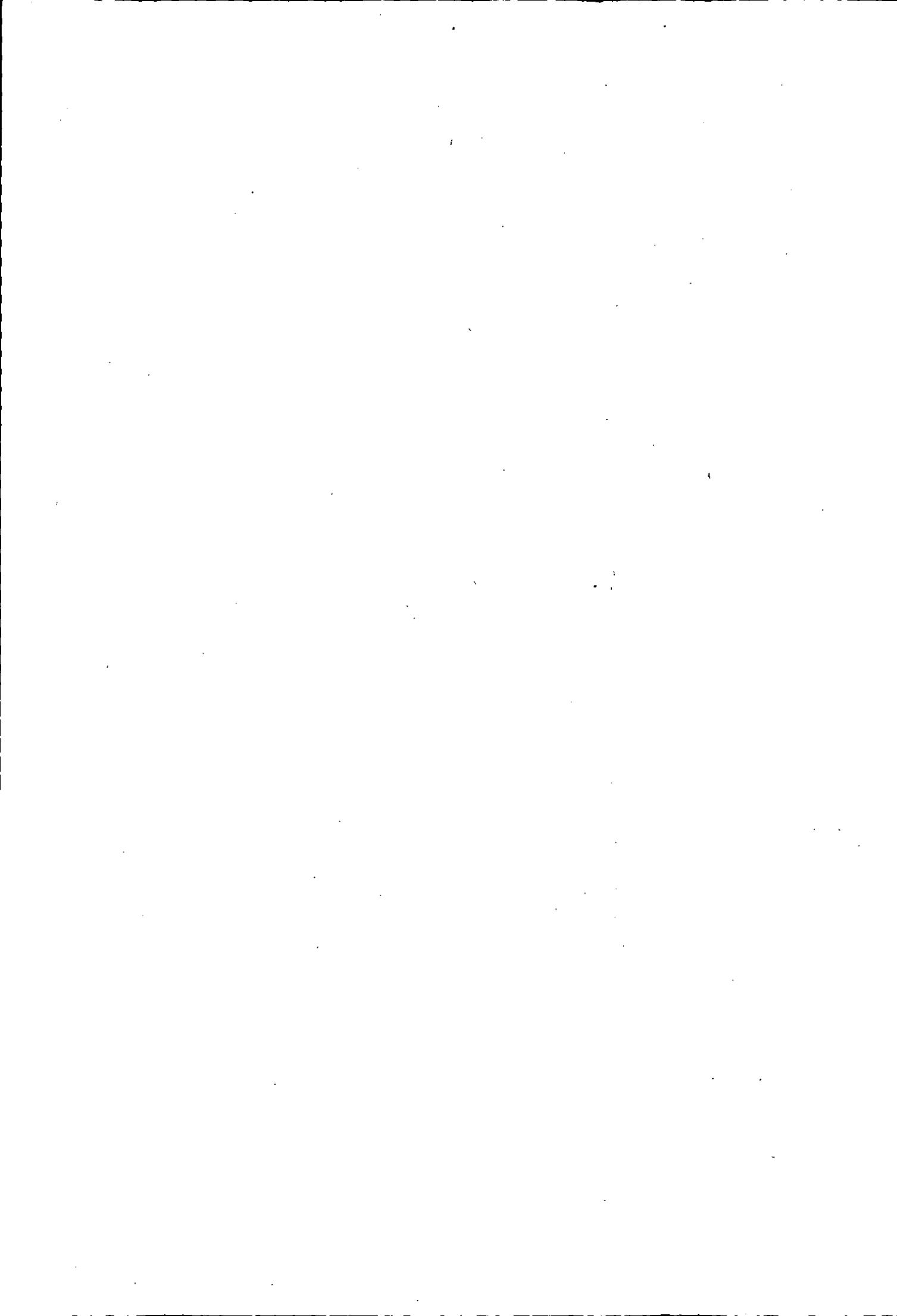


Fig. 53. Schematic load sequence in the RAE-test.



NLL-TR M. 2070

C. C. L. Class. G 042 : C 6-4

Nationaal Luchtvaartlaboratorium (N. L. L.) Amsterdam,
(National Aeronautical Research Institute), The Netherlands
PROGRAM-FATIGUE TESTS ON NOTCHED LIGHT
ALLOY SPECIMENS OF 2024 AND 7075 MATERIAL.

J. Schijve and F. A. Jacobs, 41 pages, 6 tables, 53 figures
Program-fatigue tests and constant-amplitude tests at positive mean stresses have been performed on 2024 and 7075 riveted joints in order to study the effect of the parameters of the program, the scatter under program-fatigue loading and the effect of occasional very high loads, and to check the validity of the Palmgren-Miner rule.

The results of similar investigations on notched light alloy specimens or structures have been summarized. A discussion is presented on the above mentioned objectives and the following questions: What is the usefulness of the Palmgren-Miner rule for the designer? How good is program-fatigue testing as a representation of the load-time history in service? What information may be expected from program-fatigue testing and how does it compare with the information of alternative methods?

NLL-TR M. 2070

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Nationaal Luchtvaartlaboratorium (N. L. L.) Amsterdam,
(National Aeronautical Research Institute), The Netherlands
PROGRAM-FATIGUE TESTS ON NOTCHED LIGHT
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